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DISCUSSIONS
APPLICATIONS FOR ADMISSION AND TRANSFER

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MEMBERSHIP

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

FLOW OF WATER AROUND BENDS IN PIPES[1]

By David L. Yarnell[2], and Floyd A. Nagler[3],
Members, Am. Soc. C. E.

Synopsis

In condensed form, this paper presents the more outstanding results of a series of experiments on the flow of water around bends of various shapes and various degrees of curvature in 6-in. pipes. Only the most salient points are discussed. These experiments are believed to be unique in that, with the same quantity of flow, the effect on the loss of head resulting from unequal velocity distribution in the pipe approaching the bend was fully investigated.

The experiments show: (1) That it is possible to have conditions such that the resistance to flow may be very small or unusually large in the same pipe bend carrying identical quantities of water; (2) that in a standard 90°, 6-in. pipe bend, for the same quantity of flow, with high velocity on the inside, and low velocity on the outside, of the approach pipe, the loss of head may be four times as much as would be measured in the same bend when high velocity exists on the outside, and low velocity on the inside, of the tangent leading to the bend; (3) that present formulas for computing loss of head due to bends appear to apply only to cases in which approximately uniform velocity distribution exists in the approach pipe; (4) that the losses of head in the bends experimented upon appear to vary as the square of the velocity, and not as the 2.25 power as suggested by some writers; (5) that a pipe bend may be as useful as any other device for the measurement of discharge; (6) that the direction of flow of the secondary currents in pipe bends depends entirely upon the velocity distribution in the approach pipe; and (7) that the same fundamental laws of flow through bends apply to both closed conduits and open channels.

[1] Presented at the Joint Meeting of the Power Division of the Society and the Hydraulics Division of the American Society of Mechanical Engineers at the Annual Convention, Chicago, Ill., on June 29, 1933. Discussion on this paper will be closed in November, 1934 *Proceedings*.

[2] Senior Drainage Engr., Bureau of Agri. Eng., U. S. Dept. of Agriculture, Iowa City, Iowa.

[3] Prof. of Hydr. Eng., Univ. of Iowa, Iowa City, Iowa. Professor Nagler died on November 10, 1933.

Introduction

The constant efforts of designers of pumping plants and hydro-electric plants to increase the efficiencies of their operation requires continuous research in hydraulics. Any change in design that may reduce the loss of head in the pipes and bends means increased efficiency. A bend acts as an obstruction to flow, causing additional loss of head in the case of both open and closed conduits. Certain types of bends cause internal changes in velocity to such an extent that the efficiency of the bends and down-stream channels as hydraulic conduits is considerably reduced. As a rule, disturbed flow means inefficient flow, and the maximum usefulness of the entire cross-section of the channel is not realized.

The studies have developed new data on velocity and pressure changes in bends and in the tangents adjacent. The data will be useful to designers of new plants and to engineers engaged in making plant-efficiency tests, because they show the effects that may be expected when piezometric connections are made at different points on a conduit. The data also demonstrate the necessity of taking into consideration the disturbing effects of conduit bends upon the performance of the down-stream tangents. Such conditions are often encountered in penstocks and spiral casings approaching water turbines as well as in draft-tube conduits of the elbow type.

Available manuscripts show that hydraulic research on bends has engaged the attention of engineers for more than 150 years. Apparently, the first tests were made by du Buat[4] about 1786. These tests were on pipes 1 in. and 2 in. in diameter. Numerous other investigators conducted experiments on pipe bends during the succeeding years. One of the first discussions on flow of water in open-channel bends was written by James Thomson, in England, in 1876.[5] The first field experiments on open-channel bends appear to have been made by M. Leliavski[6] on the Dnieper River, at Kiev, Russia, in 1892.

The investigations described herein were undertaken for the purpose of determining the laws governing the changes in pressure and velocity in different parts of the flowing stream, as the moving water undergoes the transition from motion along a straight path to motion around a curve, and, again, as it undergoes the opposite transition back to final straight-line motion. This condition of transitional flow also may be considered as representative of what always occurs whenever water flows in a bend or a crooked channel, or whenever it meets a bridge pier or other form of obstruction. A knowledge of these laws is fundamental to an understanding of the effects, harmful or otherwise, of crooked and obstructed channels on the flow of water, and to a determination of the best means for alleviating or relieving any objectionable or troublesome effects.

A novel feature of the investigation was the use of transparent material for the walls of the testing channel. This type of construction enabled the

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[4] "Principles Hydraulique," par E. du Buat, Paris, 1786, Pt. 1, pp. 141–151.

[5] *Proceedings*, Royal Soc. of London, May 4, 1876.

[6] "Currents in Streams and the Formation of Stream Beds," by M. Leliavski, Sixth International Congress of Internal Navigation, The Hague, The Netherlands, 1894.

observation of the direction of secondary currents by means of threads, color, and floats that could readily be seen and photographed through the exterior walls of the conduit.

Laboratory Investigations

The first tests were made on a 180° bend of square cross-section, 10 by 10 in., with a 5-in. inner radius. This shape was tried first because it seemed easier to build of transparent material, and because of the convenience of dividing the channel into elementary strips for studying flow conditions. The approach tangent was 25 ft long and the discharge tangent, 28 ft. The bend and 8 ft of the two tangents adjacent to it were made of transparent celluloid. Velocity traverses were made at eight different sections on the bend and at eleven sections on the two tangents. In each test velocities were taken at more than 800 different points for each quantity of flow. Pressures were measured at 296 different points. About 48 man-hr were required to get all the measurements for a single test.

Tests were next made on 180° bends of rectangular cross-section, 5 in. wide by 10 in. deep. One bend had a 5-in. and the other, a 10-in., inner radius. Tests were made on the bends flowing full and partly full, and with both uniform and non-uniform velocity distribution in the channel approaching the bend.

The next set of experiments was made on bends of circular cross-section, 6 in. in diameter. The bend (standard elbows with a $5\frac{1}{4}$-in. inner radius), approach, and discharge tangents were made of transparent celluloid. Tests were made on a 180° bend continuous in one direction (Fig. 1), a standard

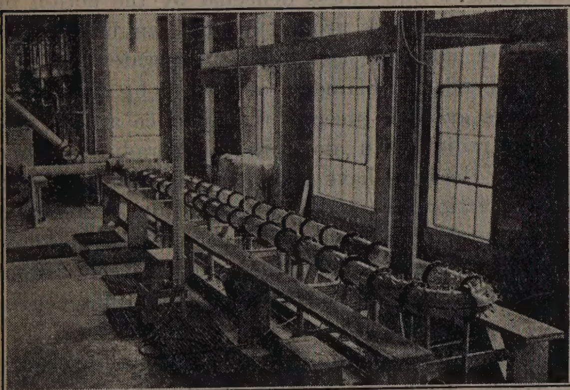

Fig. 1.—Installation of 6-Inch Celluloid Pipe for Test of 180° Bend.

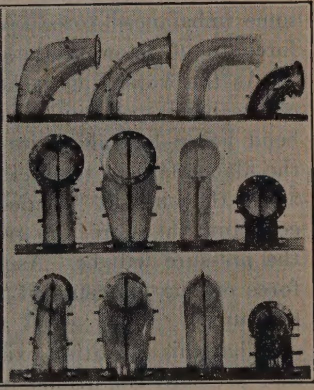

Fig. 2.—Types of 90° Celluloid Bends Tested.

90° ell, an abrupt 90° elbow, a 180° reserve curve bend, and a 270° bend made up of three 90° bends all in a horizontal plane, but twice reversing the direction of curvature. The approach and discharge tangents were each 25 ft long.

Bends of irregular cross-section were also included in the experiments. These bends included the hyperbolic and elliptical shapes as well as a circular

bend with varying radius of curvature. Fig. 2 shows the front, side, and rear views of four specimens. The one at the right is a standard elbow and the other three are special designs made by Charles A. Mockmore, M. Am. Soc. C. E., for studies on draft-tubes.

Velocity measurements were taken at seven different points on each of four diameters at each section. There were nine velocity sections on each of the approach and discharge tangents, in addition to velocity traverses at intervals of 22°.5 on the bend. For each quantity of flow, velocities were taken at 200 different points, and pressure readings were taken at about 230 different points, in the bend and on the tangents.

Velocity and pressure measurements were recorded for various flows in the pipe with a normal uniform distribution of velocity in the approach tangent. A series of tests having non-uniform velocities approaching the bend was also made. These tests consisted of a high velocity at one side and a low velocity at the opposite side of the pipe. A test was then made with the high velocity at the top of the pipe. Next, this high velocity was created at the inside of the pipe, then at the bottom of the pipe, and, finally, outside the pipe.

It is impractical to present, in a limited space, all the data obtained in the investigation.

Velocity Conditions Within the Bend

When water flows in a straight conduit or channel, the transverse water-surface profile is presumably a straight level line. On a curve or bend, due to centrifugal force, the transverse water-surface profile cannot be level. Water naturally moves in a straight line unless deflected by the action of some unbalanced force. When water moves around a bend, an unbalanced force directed toward the center of curvature acts against the water.

As the water approaches the bend, the filaments of flow along the inside of the bend speed up because the grade is steeper, the distance around the bend being less along its inner radius than along its outer radius. Thus, the filaments next to the inside of the bend have a greater velocity than those along the outside. Neglecting friction, the total energy in any filament is constant; therefore, the greater the velocity along the inner radius, the lower the pressure will be. As the water moves around the bend, the centrifugal force on the filaments of water will be balanced by the radial difference in pressure.

That this condition exists may be seen in Fig. 3, which shows the velocity and pressure conditions in a 180° bend. At Section 0°, the beginning of the bend, the filaments of flow at the inside, have already acquired a velocity considerably greater than that along the outside, of the bend. The greatest velocity occurs at a point next to the inside wall, approximately midway around the bend. The thread of maximum velocity in the channel gradually moves to the outside of the bend as the water travels around, as may be seen at Section + 0.43 ft in Fig. 3. For a given discharge, the sharper the radius of curvature the greater the velocity along the inner radius.

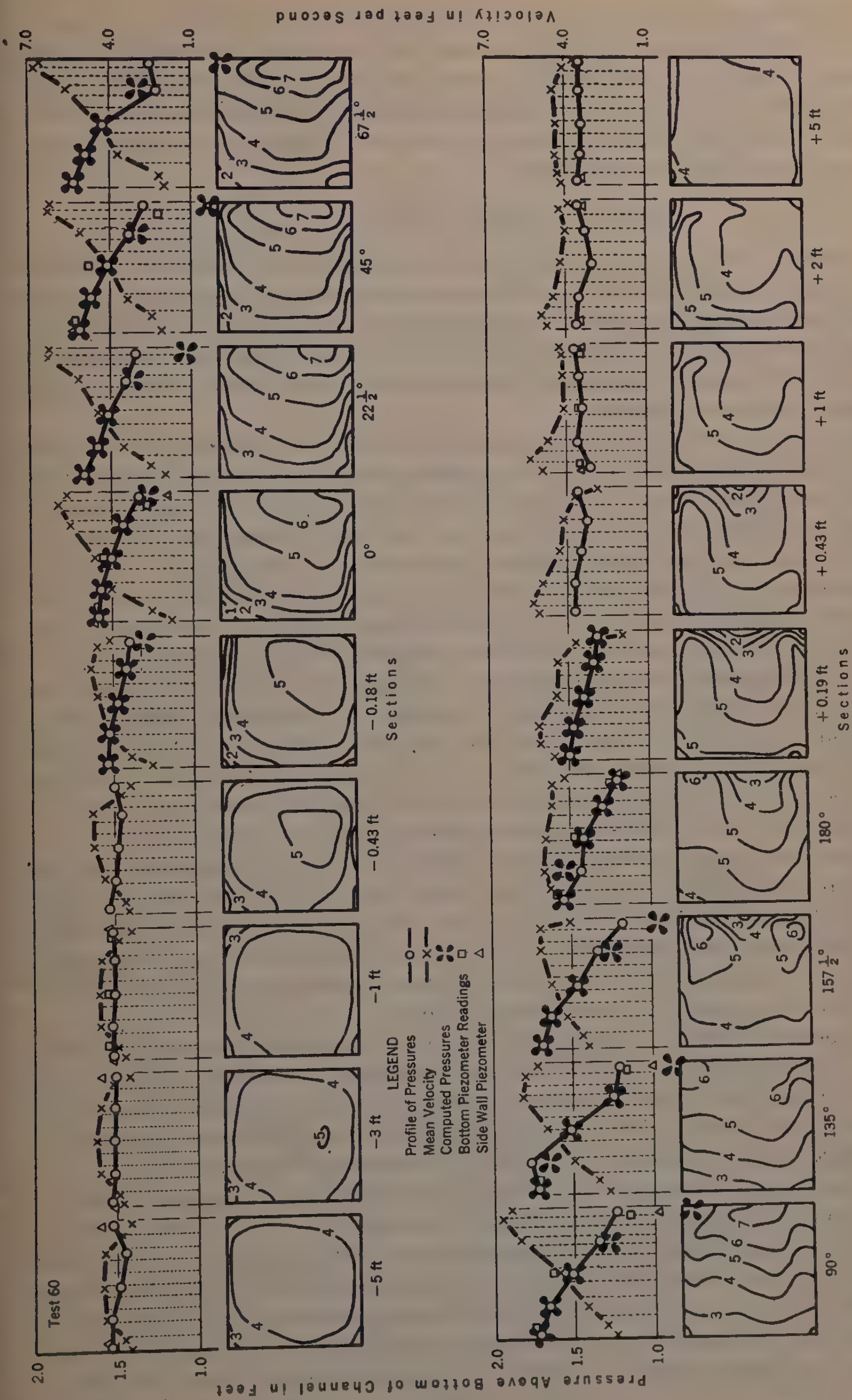

FIG. 3.—VELOCITY AND PRESSURE CONDITIONS IN 180° BEND.

The velocity distribution in circular conduit bends is similar to that in bends with channels of either square or rectangular cross-section. The velocity in a 6-in., 90° bend is shown in Fig. 4. Attention is called to the shifting of the high velocity from the inside to the outside at the end of the bend. The typical section, Fig. 4, shows the location of piezometer connections and Pitot tube readings.

Pressure Changes Within the Bend

When a stream of water meets an obstruction that diverts the various particles of water from motion along a straight line to motion in a curved path, the speed of the particles of water must be changed by unequal amounts; that is, the velocity of some parts of the water must be increased and that of other parts must be decreased. Accompanying these changes of velocity are related changes in pressure.

The pressures against the top and bottom of the rectangular channels flowing full are shown in Fig. 3 as circles connected by solid lines. Since flow around bends follows the law of free vortex motion, the transverse pressure of profile is convex upward, and has the greatest slope on the convex or inner bank and the flattest slope on the concave or outer bank. It will be noted that as the water moves around the bend the pressure is greatest near the outer side of the bend and is much lower along the inner side. This condition was true even when the channels were flowing only partly full.

The peripheral pressures at various sections around bends of square and rectangular cross-sections are very enlightening. Peripheral pressures were measured at Sections − 1.0 ft, 0°, 45°, 90°, 135°, 180°, and + 1.0 ft on the 10 by 10-in. 180° bend. Pressures were measured at twenty-two different points on the periphery for various quantities of flow, seven on each of the outside and inside walls, three on the bottom, and five on the top.

It will be noted that the pressure, measured as above the channel bottom, is uniform on the four walls of the conduit in the approach channel to the bend (Section − 1.0 ft (Fig. 5)). As the water moves around the bend the pressures on the inside wall of the bend decrease, as shown in Sections 45°, 90°, 135°, and 180° (Fig. 5). The pressure against the outside wall of the bend at any one section is practically uniform.

When non-uniform velocities prevail in the channel approaching the bend, the periphery pressures within the bend become much more irregular, as may be seen in Fig. 6. In this case the bend was 5 in. wide by 10 in. deep, with a 5-in. inner radius. Attention is called to the large differences in periphery pressure at the 45° section, as shown by comparing Figs. 5 and 6.

The pressure distribution in circular conduit bends is similar to that in bends on either square or rectangular cross-section (see Figs. 3 and 4).

The pressure differences between the inner and outer sides of a 90° bend in a 6-in. pipe, on horizontal diameters, are shown in Fig. 7. Pressure differences are shown for five different conditions of velocity distribution in the approach pipe to the bend, for a mean velocity of 8.3 ft per sec. Tests show that, for a given velocity distribution approaching the bend, there is a marked

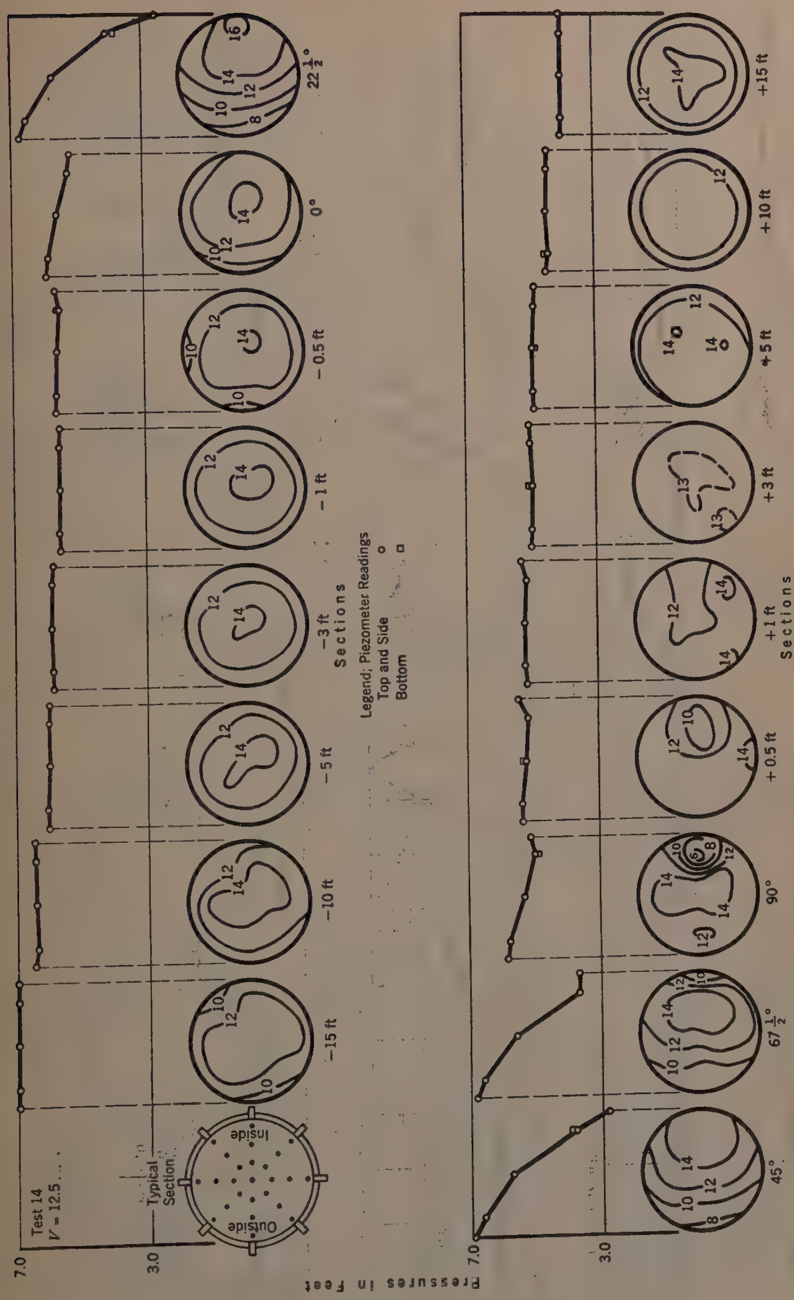

FIG. 4.—VELOCITY DISTRIBUTION AT VARIOUS SECTIONS IN 90° BEND IN 6-INCH PIPE (RADIUS OF CENTER LINE 8¼ INCHES; MEAN VELOCITY, 12.5 FEET PER SECOND; APPROXIMATELY UNIFORM DISTRIBUTION IN APPROACH TANGENT; TRANSVERSE PROFILE OF PRESSURE SHOWN ABOVE EACH SECTION).

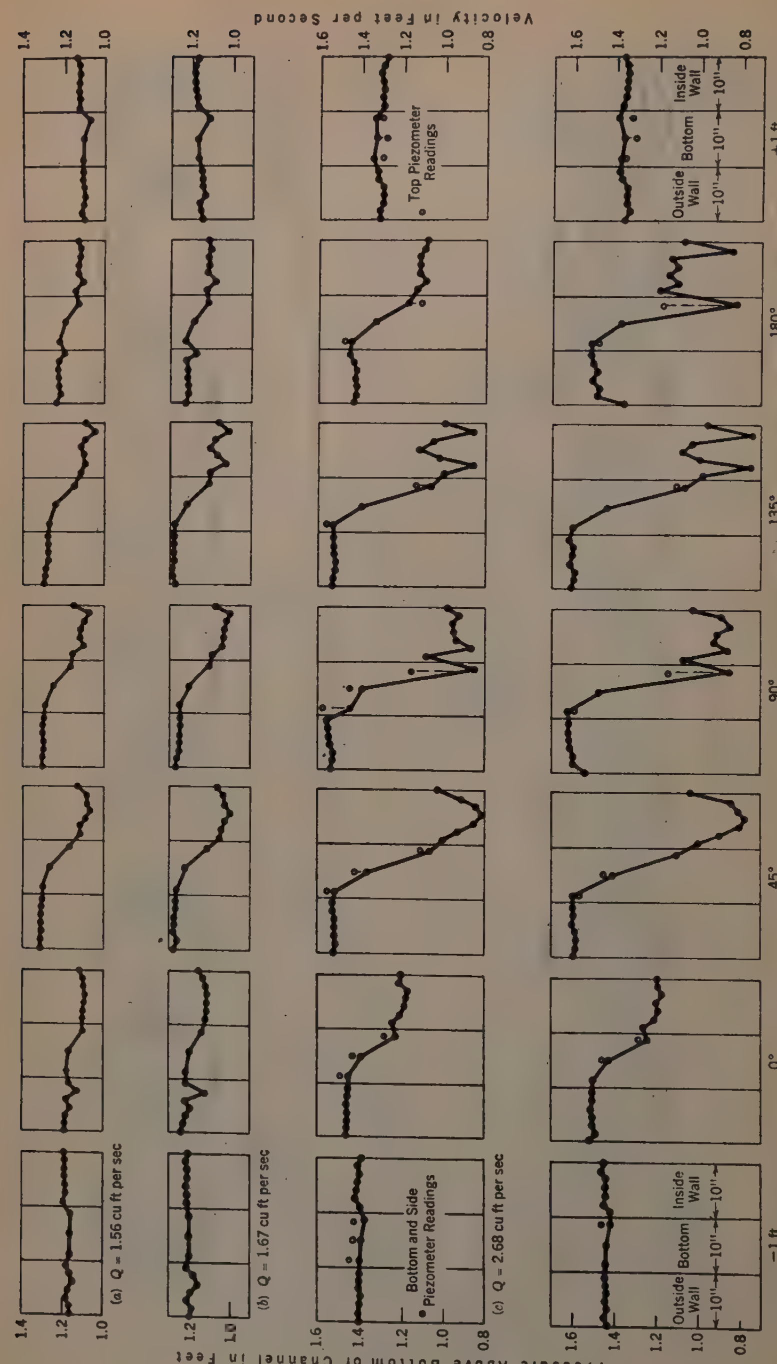

FIG. 5.—PRESSURES ON PERIPHERY OF 10-INCH SQUARE CHANNEL ON 180° BEND OF 5-INCH INNER RADIUS, WITH UNIFORM DISTRIBUTION OF VELOCITY IN APPROACH CHANNEL (FLOW, 1.56 TO 2.90 CUBIC FEET PER SECOND).

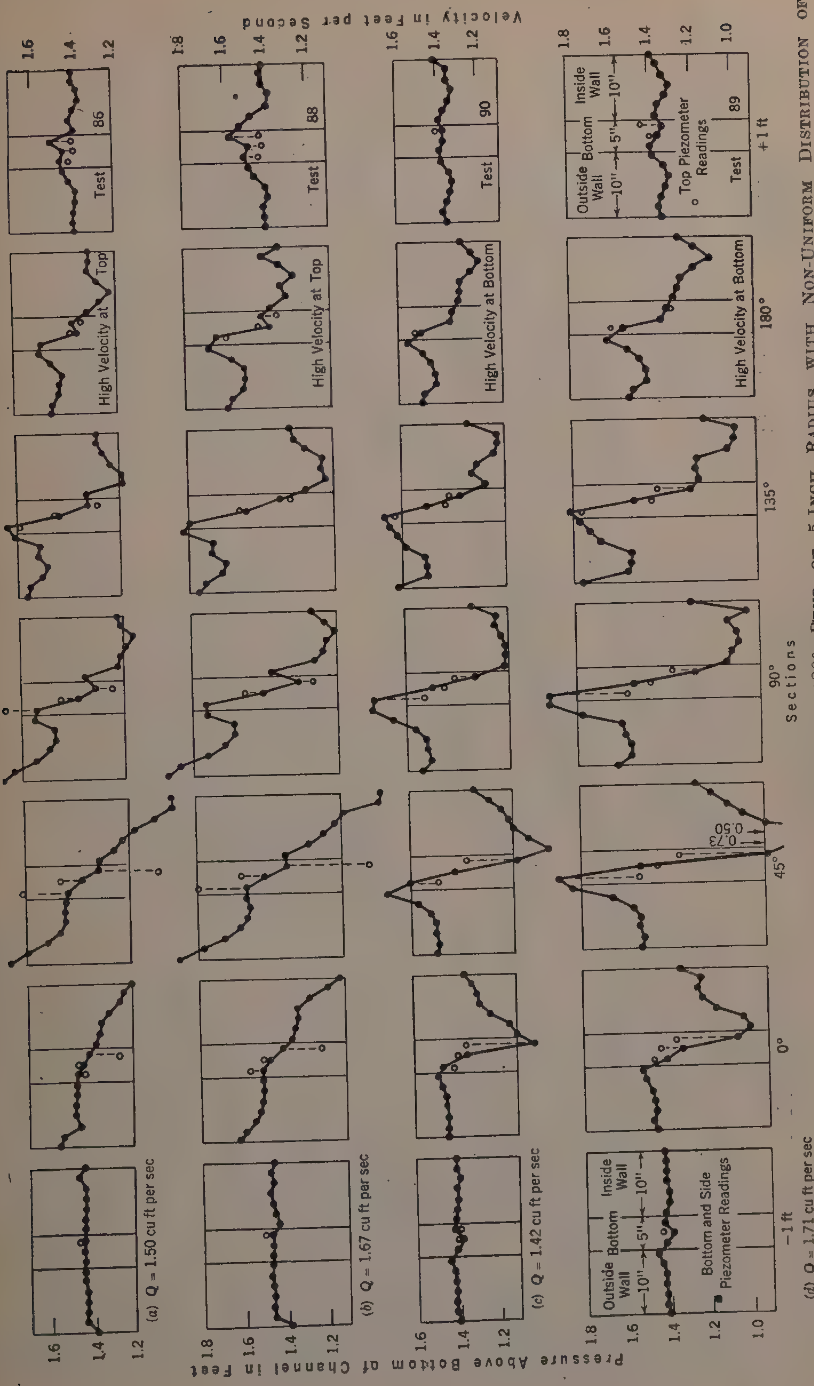

FIG. 6.—PRESSURES ON PERIPHERY OF 5 BY 10-INCH CHANNEL ON 180° BEND OF 5-INCH RADIUS WITH NON-UNIFORM DISTRIBUTION OF VELOCITY IN APPROACH CHANNEL AS SHOWN (FLOW, 1.50 TO 1.71 CUBIC FEET PER SECOND).

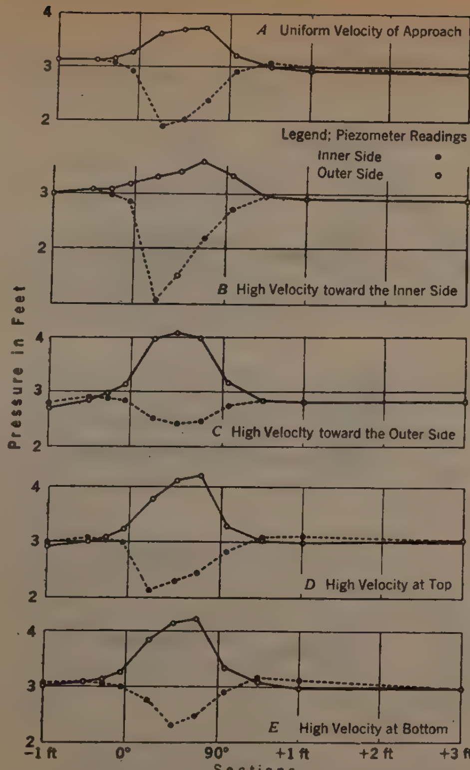

FIG. 7.—PRESSURES ALONG INNER AND OUTER SIDES OF 90° BEND IN 6-INCH PIPE ON HORIZONTAL DIAMETER, WITH VARIOUS VELOCITY DISTRIBUTIONS IN APPROACH CHANNELS. (RADIUS OF CURVATURE OF CENTER LINES 8½ INCHES; FLOW FROM RIGHT TO LEFT; DISTANCES MEASURED ON CENTER LINE.)

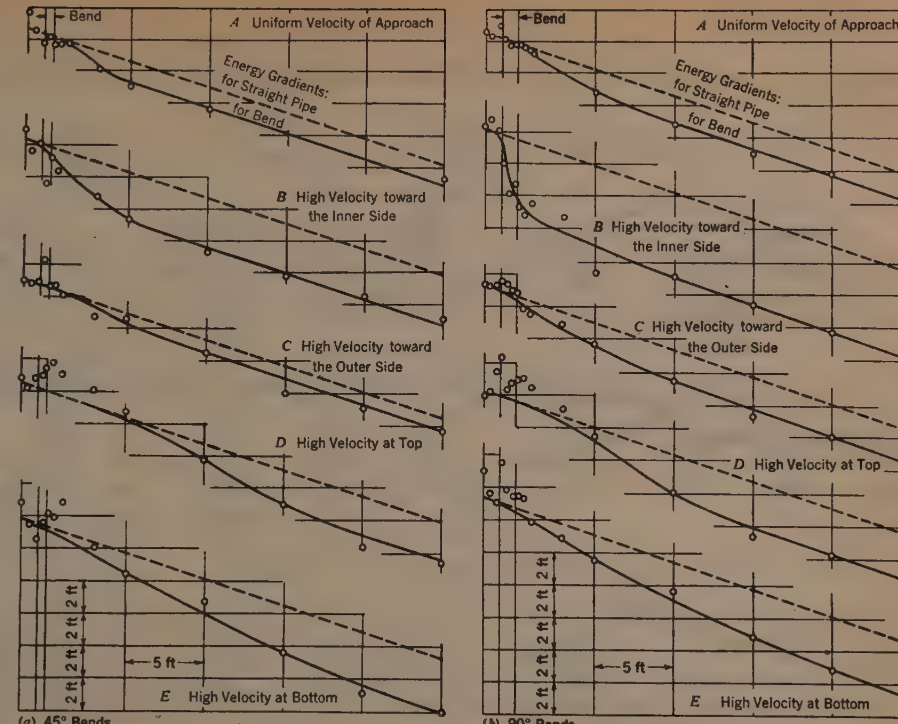

FIG. 8.—SECTIONS SHOWING LOSSES OF HEAD IN 45° AND 90° BENDS IN 6-INCH PIPE FOR DIFFERENT VELOCITY DISTRIBUTIONS.

similarity of pressure differences near the beginning of the bend, regardless of the degree of curvature. In bends of square and rectangular cross-section, the longitudinal pressure curves are similar to those obtained with circular pipe bends.

Differences in pressure between the inside and outside of pipe bends may exceed several feet of water column even with ordinary velocities. With a mean velocity of 10 ft per sec in a 6-in. pipe, a 90° elbow caused a difference of about 6 ft of water column, but the addition of another 90° elbow did not increase this difference. A 6-in., abrupt 90° ell gave a difference of more than 10 ft, with a mean velocity of 10 ft per sec.

Knowledge of these differences of pressure in bends is of importance to engineers engaged in pump efficiency tests, particularly in plants in which gauge readings in the suction and discharge pipes are used to determine the total head pumped against. If a gauge must be attached to a bend, it is desirable to have four piezometric connections made at right angles to each other and carried to a common manifold to which the gauge is attached.

Loss of Head in Bends

The loss of head caused by a pipe bend depends on several factors, a very important one of which is the kind of velocity distribution that occurs in the pipe approaching the bend. It is possible to have such conditions of flow that the loss of head may be very little or unusually large in the same bend for identical discharges. Fig. 8 (*a*) shows the losses in 45°, and Fig. 8 (*b*), the losses in 90°, bends in 6-in. pipe for different velocity distributions in the pipe approaching the bend, with a quantity of flow giving a mean velocity of 8.3 ft per sec. This unequal velocity distribution was created by a special screen placed 5 ft up stream from the beginning of the bend. The obstruction gave a low velocity of about 4 ft per sec, and a high velocity of 12 ft per sec, at the beginning of the bend.

With the velocity in the approach pipe high toward the inner side, and low toward the outer side, of the bend, the loss of head may be two to four times as much as would be measured in the same bend with uniform velocity distribution existing in the approach pipe. On the other hand, with the approach velocity high toward the outer, and low toward the inner, side of the bend, the loss of head may be less than that caused by the same bend when uniform velocity distribution prevails in the approach tangent. With high velocity either at the bottom or at the top in the approach, the loss of head caused by a bend is considerably more than would occur if uniform velocity distribution prevailed in the approach tangent.

The data obtained show that the present formulas for computing loss of head in bends apply when uniform velocity distribution exists in the pipe approaching the bend.

The loss in a 45° bend (see Fig 8 (*a*)) is about three-fourths that in a 90° bend (Fig. 8 (*b*)) of the same radius. The loss in a 90° abrupt elbow is about 1.2 times the velocity head, or about 7.5 times that in a standard 90° elbow.

Secondary Currents in Bends

Secondary currents within the bend are caused by the difference in the centrifugal forces of the filaments of varying velocity; that is, transverse flow occurs in a pipe or channel bend in addition to the forward motion of the fluid. These induced, or secondary, currents usually originate in that section of the pipe next to the top and bottom and gradually increase in strength and intensity as the water moves around the bend.

The types of secondary currents that prevail in a bend depend on the velocity distribution in the approach tangent. The unequal pressures against the inside wall of the 180° bend of square section (Sections 135° and 180° (Fig. 5)), are caused by the secondary currents set up within the bend. If a high velocity occurs at the top, and a low velocity at the bottom, of the approach channel, the secondary current will have a counterclockwise motion, as viewed looking down stream, in a conduit that bends to the right; if the high velocity is at the bottom in the approach channel, the direction of the secondary current will be clockwise. If uniform velocity distribution exists in the approach pipe, two secondary currents will be set up in the bend, one clockwise in the upper portion and the other clockwise in the lower portion, of the cross-section. In Fig. 9, the yarns next to the outside wall

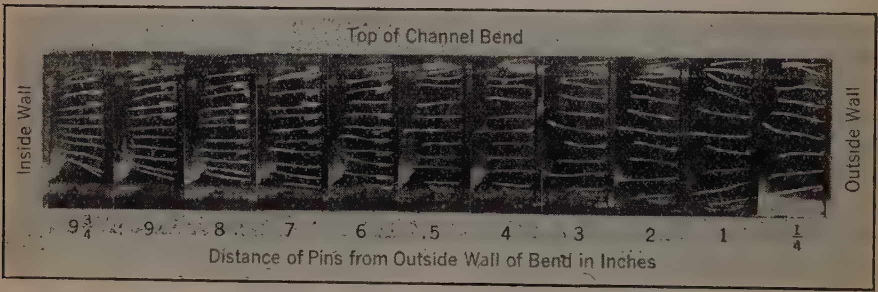

Fig. 9.

diverge, while those next to the inside wall converge. Secondary currents exist also in open-channel bends, but usually there is only one, even with uniform velocity distribution in the channel approaching the bend.

The loss of head in bends is due primarily to two causes: (1) Skin friction of the forward flowing water against the walls of the pipe; and (2), internal friction of the induced or secondary currents within and following the bend. The strength of the secondary currents in bends is greater in wide channels than in narrow channels. Hence, the practice of using blade turns in 90° elbows and guide-vanes in quarter-turn draft-tubes greatly increases the efficiency of these bends, due to reduction in the magnitude of the secondary currents.

In general, the greater the velocity, the more pronounced will be the secondary currents in bends, and any means devised to reduce these currents and to create a greater "effective area" of forward flow will reduce the energy loss and hence improve the efficiency.

Pipe Bends as Flow Meters

It would appear that advantage might be taken of the differences in pressure between the outside and the inside of a pipe bend, and that the bend could be used as a flow meter. The difference in pressure between the inside and outside of a bend may be fully as large as, or even larger than, that produced in a Venturi meter or Pitot tube and, as in those other measuring devices, it varies with the second power of the velocity. The experiments indicate that the errors likely to be involved are no greater than those experienced in these other devices. Each bend, however, should receive individual calibration where great accuracy is desired.

It can readily be proved that the difference in elevation of the water surface between the two sides of the bend is equal to:

$$\frac{V^2 B}{g R} \quad \text{.......................... (1)}$$

in which, V is the mean velocity at the section; B, the width of that section; g, the acceleration due to gravity; and R, the mean radius.

The formula, $H = \frac{V^2 B}{g R}$, can be written for all pipe bends, as follows:

$$V = c \sqrt{H} \sqrt{\frac{g R}{B}} \quad \text{.......................... (2)}$$

in which, c is a coefficient depending on the kind of pipe. The tests with 6-in. celluloid pipe gave a value of c of about 0.91 for the 90° bend and a few tests with 6-in. cast-iron pipe also gave a value of c of about 0.91 for the standard 90° elbow. The pressure difference in the celluloid pipe was taken at 22°.5, and in the cast-iron pipe, 21°.5, from the beginning of the bend. It would appear that Equation (2) would give very close results in determining the quantity of flow. Further tests will be made on bends to ascertain the variation in the coefficient with the variation in position of measuring the pressure difference.

Equation (2), simplified for a single pipe bend, may be written as:

$$V = c' \sqrt{H} \quad \text{.......................... (3)}$$

in which, H is the velocity head in the pipe, and c', a constant. For the celluloid 90° bend, taking the difference in pressure at a section 22°.5 from the beginning of the bend, c' was found to be 1.14. For the 6-in., 90°, cast-iron bend, c' was 1.17, with the difference in pressure measured (of necessity) at 21°.5 from the beginning of the bend.

When unequal velocity distribution prevails in the approach channel to the bend, the best results probably would be obtained by measuring the pressure differences at the 45° point from the beginning of the bend, although further tests are needed to verify this point. For this condition the variation in c may be somewhat greater, particularly when high velocities prevail on the inside of the approach pipe to the bend unless the particular bend in question is calibrated.

New Types of Pipe Bends

Studies of the energy losses in pipe bends have resulted in the development of several new types of bends. In pipe bends carrying water, comparatively little has been done in suggesting changes of shape to reduce the loss of head. Lack of effort in this direction has been due mainly to the fact that in such pipes the velocity of flow is not always high. In pumping plants and similar units, long-radius elbows of uniform section or tapered reducing sections frequently are used. It would appear that some effort might profitably be expended in investigating and experimenting on various shapes of elbows.

Many shapes of bends are suggested, but only tests can show their good or their bad qualities. Some have suggested bends of the hyperbolic type, while still others have devised bends of elliptical shape so that the difference in velocities between the inside and the outside of the bend will be comparatively small. The difference in velocity helps to start secondary currents and the resultant inefficient flow. A new type of radius for a bend also has been suggested (see Fig. 3), in which the length of the radius at the point of tangency is infinite and decreases until, midway around the bend, it becomes a minimum, equal to the radius of the regular bend. The shape of the last half of the bend is similar to that of the first half.

The special bends shown in Fig. 3 are little better than the standard 90° bend tested, in so far as loss of head is concerned. Due to lack of funds, it was impossible to test other special shapes.

Conclusions

The following conclusions have been drawn, based on the data thus far collected in the investigations:

(1) All bends act as obstructions to flow, resulting in additional loss of head.

(2) With uniform velocity distribution in the approach channel to the bend, the velocities of the filaments along the inside wall of the bend are increased while those along the outside wall are reduced.

(3) For a given pipe bend and quantity of flow, the head lost in the bend depends upon the velocity distribution in the approach tangent.

(4) In a standard 90°, 6-in. pipe bend, for the same quantity of flow, with high velocity on the inside and low velocity on the outside of the approach pipe to the bend, the loss of head may be four times as much as would occur in the same bend when high velocity occurs on the outside and low velocity on the inside of the tangent leading to the bend.

(5) Knowing the maximum difference in pressure between the inside and the outside of the bend, and the radius and size of the bend, it is possible to compute the mean velocity in the pipe, and, therefore, the discharge.

(6) After a pipe bend has been calibrated, it may be used as a flow meter and discharges may be determined by measuring the difference in pressure between the inside and outside of the bend.

(7) The losses in the pipe bends experimented upon appear to vary as the square of the velocity and not as the 2.25 power as suggested by some writers.

(8) The fundamental laws that apply to closed conduit bends, also apply to open channel bends.

ACKNOWLEDGMENTS

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AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

STREET THOROUGHFARES
A SYMPOSIUM

PAGE

*Note.—Discussion on this Symposium will be closed in November, 1934, *Proceedings*.

FOREWORD

By George H. Herrold,[1] M. Am. Soc. C. E.

In July, 1930, a Division Committee on Manuals was re-organized and the general subject of a City Planning Manual was discussed at a Division meeting in Cleveland, Ohio. It was decided that there should be a series of committees to study and report on the various subjects involved in such a manual and it was agreed that the first report to be completed should deal with Street Thoroughfares.

A Committee on Street Thoroughfares Manual was created by the Executive Committee of the City Planning Division on January 22, 1931. The Committee began active work in the spring of that year and developed a program to produce definite results within a reasonable time. A progress report of the Committee was presented at the meeting in Tacoma, Wash., in July, 1931, and included a bibliography and definitions. At this same meeting papers were also presented on allied subjects; that is, "Population Density Related to Roadway Area," "By-Passing Traffic Around Small Towns," and "Increasing Highway Efficiency."

Since this date the Committee has been actively engaged in the preparation of this Symposium. Its purpose is to suggest methods of identifying street thoroughfares according to their use and capacity and of outlining their design according to the best usage and well-defined trends of the present day.

The word, "thoroughfare," may be given the broadest possible interpretation of all words meaning a place for the movement of commodities and people.

In 1386, Chaucer wrote: "The world is nothing but a thoroughfare", and in Shakespeare's "Henry the VIII" (1540), is found "Chauncerie Lane and Pewter Lane being thoroughfares and passages from Fleet St. into Holborne." Dalton's "Country Justices" (1630) states: "In towns which are not thoroughfares the justices shall be sparing of allowing any ale-houses," and in Cooke's "Voyages in the South Seas" (1712): "There is a thoroughfare in the midst of it where we rode with our ships." In Emerson's "English Traits" (1856), is the expression: "They have made the island a thoroughfare and London a shop inviting to strangers" and, quoting from the *Westminster Gazette* of June, 1908: "How seldom must these ancient walled villages communicate with the thoroughfare valleys."

A street thoroughfare, therefore, is a way through a street open at both ends, through which there is much passing. The place of the street thoroughfare in the city plan structure is the most important. It affords accessibility to land and makes it usable. It is a definite part of the traffic network of State, region, or city. It drains the down-town district of vehicles quickly at the close of the business day or after the theatre. It is the avenue connecting the home districts with the business district. It is the driveway through parks or park reservations—a thoroughfare for recreation and

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pleasure. It is the street over which commodities and people are moved. It makes living agreeable and business possible.

The street thoroughfare is created by the necessity of travel between terminals. It determines the uses of land along its course. It forms the pattern that moulds the city. It lies between the origin and destination of traffic. It is a right of way for tree planting—a thoroughfare glorified by landscaping. It is a right of way for subterranean or underground utilities. Street thoroughfares dominate the city plan by determining the controlling major or minor traffic movements. The conflicting demands of "access to property" and "continuous traffic movements" are supplied by it. It is the *ne plus ultra* for ground movements. It connects terminals and, at the same time, serves abutting property. It creates the mosaic of city, region, or State, and becomes the radial, the boundary, or the axis of planned and zoned areas.

The many miles of frontages along street thoroughfares both in urban and suburban territory present peculiar and difficult problems in planning their use. The passing traffic is increasingly of a character that requires no service from abutting property between terminals, leaving much land valueless for speculative ventures or business serving a traveling public. In suburban areas the land retaining its agricultural use is enhanced in value only by the shortening in time to a market, and reduced in value by excessive traffic and traffic hazards.

As the principal use of land in urban areas is for streets, parks, and building sites, the location of street thoroughfares in their proper relation to each other and to building sites and open spaces is the first element in city planning. The street thoroughfare includes all trafficways, until further defined and restricted—the lane, the avenue, the mercantile street, the boulevard, the express-way, etc.

It is hoped that this Symposium will assist in clarifying some loose thought and discourage careless writing through the specifications set up by it, creating a terminology that may be used by all city planners.

DEFINITIONS, GENERAL PRINCIPLES, AND RECOMMENDATIONS

REPORT OF THE COMMITTEE OF THE CITY PLANNING DIVISION ON STREET THOROUGHFARES MANUAL

W. W. CROSBY,[2] M. AM. SOC. C. E., *Chairman*

The Committee on Street Thoroughfares Manual, appointed on January 22, 1931, herewith submits the following report in which it has quite thoroughly considered a relatively new subject. After the results have become more generally known and discussed, and after some future developments have occurred in street-thoroughfare practices, a further reconsideration of the subject may be productive of more valuable and specific conclusions.

The work of the present Committee—widely scattered in its membership—has had to be conducted entirely by correspondence. However, by sending, contemporaneously, copies of all texts to each member, all decisions have been made by an equally advised membership, and finally have been unanimous. Except for the individual papers, the report text is that of the Committee as a whole.

To illustrate the operations of the Committee, the following outline may be given: After acceptances of membership on the Committee had been secured, suggestions as to its objectives were obtained from Messrs. Harold M. Lewis, Secretary of the City Planning Division, and George H. Herrold, Division Sponsor for the Committee. Pursuant thereto, Jacob L. Crane, Jr., Jay Downer, W. W. Horner, and Daniel L. Turner, Members, Am. Soc. C. E., were requested to prepare their respective papers and to define certain terms. The Chairman, on receipt of these responses, collated the definitions and resubmitted them with some others for discussion. John P. Hallihan, M. Am. Soc. C. E., was added to the Committee membership in August, 1931, and his paper was received soon after the request then made for it. He also contributed valuable suggestions for definitions.

The discussion concerning the definitions was prolonged and has now finally resulted in the expressions given.

Progress reports were submitted at the meetings at Tacoma, Wash., in July, 1931, and at St. Louis, Mo., in October, 1931. During 1932, the correspondence concerning definitions, papers, and other Committee affairs was continued.

In January, 1933, the Chairman submitted to the members of the Committee (with copies to Chairman Noyes, Mr. Herrold, and Secretary Lewis) a tentative draft of the Summary and Conclusions, abstracted mainly from the papers by the members and the correspondence of the Committee. Discussion again followed until full accord was reached, as submitted.

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The Committee proposes the following definitions of terms used in its report in order that its meanings may be clear. The general adoption of these terms and definitions is recommended in the interests of clarity and brevity.

Definitions

Alley.—A street less than three traffic lanes in width, usually in the rear of building lots, and designed for local service only.

Avenue.—See *Road* or *Street.*

Boulevard.—An important street of extraordinary width (not less than 120 ft wide), with ample provisions for shade trees or ornamental planting. A boulevard is not intended for trucking but is readily available and attractive to pleasure traffic, and adjacent to property that may be peculiarly adapted to furnishing imposing settings for public buildings or for the highest type of private residences. In its design ample or generous provisions are made for pedestrians, for restful recreation, or contemplation, and for ornamentation.

By-Pass Road or Route.—A road or route providing for the passage of through traffic around, instead of through, a locality.

Freeway.—A highway to which there is no vehicular access from abutting properties.

Highway.—The entire right of way devoted to public travel, including the sidewalks and other public spaces.[3]

Arterial Highway: A highway for vehicular traffic that is a main channel, with many tributaries, and the roadway of which is required, by traffic frequency, to be not less than four traffic lanes wide.

Express Highway: A highway having an express roadway.

Express Roadway: A roadway for fast minimum-stop traffic, with separated opposite-direction free-traffic lanes, and without grade crossings.

Express Traffic: Non-stop, fast traffic with distantly separated origin and destination.

Expressway: A highway for express traffic exclusively.

Major Highway: A highway forming an essential part of a highway system for a region, such as a State, and the right of way of which is not less than 100 ft wide.

Secondary Highway: (1) A highway with right of way less than 100 ft wide, tributary to, or parallel with, and of less importance than, a major highway; (2) a highway with right of way less than 100 ft wide.

Super-Highway: A highway to accommodate mass passenger transportation on rails, in addition to all other highway functions, including local service to abutting properties.

Lane.—A narrow secondary highway. A strip of roadway passed over by successive vehicles traveling in the same direction, including in its width an excess on both sides for the clearance of passed vehicles.

Loading Lane: The strip of roadway reserved, or customarily used for, the loading of vehicles.

[3] *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1425.

- Parking Lane: The strip of roadway reserved, or customarily used for, the parking of vehicles.
- Standing Lane: The strip of roadway reserved, or customarily used for, standing vehicles.
- Free Traffic Lane: A lane of traffic not used for loading, parking, or standing, nor containing street railway tracks, along which traffic may proceed in one direction freely and without interruption, except by traffic signals and the usual requirements of safety.

Parking.—The standing of vehicles, attended or not, for any purpose except the observation of traffic regulation, congestion, or signals.

Parkway.—A thoroughfare, with a right of way normally not less than 300 ft wide, planted with trees or other decorative growth, where vehicular access to abutting properties is precluded except by supplementary roadways connecting with that of the parkway at widely separated intervals; and where grade crossings with main intersecting highways are eliminated.

Plaisance.—A short parkway, extraordinarily ornamented by plantations, and where parking spaces for the enjoyment of the views are provided for pleasure vehicles. A narrow park on which houses face, the street of vehicular approach to the house being in the rear.

Planting Space.—The strip or area of the highway set aside for grass, trees, shrubs, flowers, statuary, and other ornamentation. (Sometimes, regrettably and confusingly, the planting space is still referred to as "Parking.")

Road.—A highway outside an urban district.[4] This term is sometimes used to designate a meandering street discordant with the general street plan.

Roadway.—That part of the highway particularly devoted to the use of vehicles.[4]

Street.—A highway in a urban district.[5]

- Heavy Traffic Street: A street carrying more than the average traffic per square yard of roadway.
- Major Residential Street: A street through a purely residential district, so located as to be a satisfactory distributor to other residential streets of the district; it serves the district as a traffic-way in contrast to serving merely the block unit; and usually requires a roadway width of not less than three traffic lanes.
- Mercantile Street: A street devoted to local commerce.

Street Thoroughfare.—A street carrying through traffic from and to other than the abutting properties.

Traffic.—The movement of goods and persons on the highways.

- Fast Traffic: Motor traffic proceeding continuously at speeds greater than 40 miles per hr.
- Heavy Traffic: Traffic of more than the average per square yard of roadway, with consideration of both the number and the weights of the traffic unit.

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[4] *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1426.

[5] *Loc. cit.*, p. 1428.

General Summary

There is a general recognition of the fundamental necessity for street thoroughfares. A fairly widespread agreement also is apparent that the ordinary development of such arteries of traffic has been, and is, insufficient even when no regard for the morrow is taken into consideration. It is also evident that, unless more frequent advance planning for street thoroughfares is established, the accidental development of ordinary streets into street thoroughfares will continue to the disadvantage of traffic and with the deterioration of the adjacent property.

Furthermore, it seems that most city planners and those others more or less actually informed on such matters quite well agree that the methods of providing street thoroughfares by simply widening the existing streets or by laying out new streets of extraordinary width, sometimes do not provide the desired results except temporarily. Even then such results as are thus had may be: (1) Extravagant, in that they require tremendous expenditures proportionately; (2) insufficient, to the extent that more width and additional traffic lanes do not properly move traffic; or (3) often destructive of values in the abutting property in that, if such property is residential, it may become less desirable, and if mercantile, it may be hampered by the increased width of the thoroughfare.

In some cases, mercantile property may even be cut off from direct access to the thoroughfare. The results of widening existing streets may be abandoned in despair later, when officials in charge recognize the impossibility of further development and that express highways on other locations will have to be provided to meet the needs for the actual movement of traffic.

While each case calls for its individual solution, certain principles seem to underlie all the problems. The effort of the Committee has been to discover these principles, to suggest them in this report, and, at least, to make a beginning toward the desirable end of a general recognition and the ultimate development of these fundamentals.

Underlying Principles

Some of the "Principles" unanimously agreed on by the Committee may be stated, as follows:

Necessity for Street Thoroughfares.—

1.—Street thoroughfares are needed for: (*a*) Mass movements of general traffic, with a minimum of congestion and a maximum of speed and safety for the entire mass, as exemplified by major highways in and adjacent to a business district; (*b*) certain extra speedy traffic, with a minimum of danger; for this type, express highways are indicated; (*c*) certain leisurely traffic, with a maximum of safety and of pleasure to that traffic as accommodated on boulevards and parkways; and (*d*) all the traffic likely to use the highway, including the local traffic as well as the through traffic, with a minimum of danger or congestion and a maximum of safety and convenience to all, as in the case of major highways, or super-highways.

Segregation of Opposing Traffic and Traffic Lanes.—

2.—To accomplish the ends mentioned under Principle 1, it may often be necessary to segregate the classes of traffic and to provide separate kinds of thoroughfares under different circumstances.

3.—The segregation of traffic may even be extended to the physical separation of vehicular traffic lanes (by a planting space) on a two-way thoroughfare; or by the establishment of one-way thoroughfares, and the abolition of grade crossings where cross-traffic cannot be cared for satisfactorily by rotary or similar intersections.

4.—Traffic-separating strips on expressways may effectually be as narrow as 4 ft in width in order that sufficient of the total width of the right of way may be devoted to the protection of the highway on both sides of the double roadway. Where, however, a sufficiently wide right of way will permit, a wider separating strip may be desirable; and, where vehicular passage from one roadway to the other is to be permitted, the separating strip should preferably be as much as 40 ft wide, in order that the turning radius for the vehicle may be practicable and that the left-turning vehicle may come to a stop out of the traffic before entering the opposite main traffic lane.

5.—For express highways carrying traffic in both directions, the opposing traffic should be physically separated (by planting or otherwise) and each roadway should be of not less than two, preferably three, and not more than four, traffic lanes.

6.—A maximum of three lanes for moving vehicles is usually best for a one-way traffic roadway.

7.—Such segregation and separate provision, if not a matter of present need, should be contemplated as a possibility in the future. If possible, this should be provided for through the early acquisition of the width of right of way ultimately desirable, and then by the progressive gradual changes that may prove feasible as they become necessary.

8.—The complete segregation of pedestrian traffic—and even of street railway traffic also—from the vehicular traffic of street thoroughfares may be warranted in some cases.

Parking and Side Roads.—

9.—No standing of vehicles should be contemplated or allowed contiguous to the lanes for moving vehicles along a street thoroughfare. Any parking, even momentary, for service to adjacent property or for rest or recreation, should be provided for in a service roadway or in parking spaces off the thoroughfare roadway itself.

10.—Ingress and egress for such service roadways or parking spaces should be provided so as to interfere in the minimum degree with traffic along the thoroughfare.

Location of Street Thoroughfares.—

11.—In considering the selection or location of street thoroughfares, recognition should be accorded the possibility of satisfactory low-grade routes parallel with stream channels. Such thoroughfares could be developed in

connection with the preservation and landscaping of the drainage ways, and would assume parkway characteristics to some extent. Furthermore, the topography that suggests a location for a canal or flume between two points will indicate even more strongly a route for a heavy traffic highway.

There are many other advantages in utilizing for thoroughfares, land along a watercourse, and proper regional planning should provide a system of thoroughfares bordering on reserved natural drainage ways.

12.—The highest function of a street thoroughfare is to supplement other traffic routes and thus to provide for satisfactory transportation not otherwise obtainable. This function may not be the only one demanded of a street thoroughfare, but the optimum results will be had when that function is not subordinated to, or seriously interfered with, by others.

13.—All highways users enjoy pleasing surroundings and, other things being equal, will tend to congregate on the more pleasant thoroughfares.

14.—Inadequate traffic solutions result only in impasses such as are everywhere to be found in cities that now realize they have more to do than they can pay for to relieve their traffic situations. Careful consideration of the relationship between practicable street thoroughfares and other arteries of transportation, with the highest kind of trained pre-vision in such matters, is necessary to prevent a constant repetition of such inadequate solutions.

Cross-Traffic and By-Passes.—

15.—Where traffic across a freeway must be provided for at grade, a possible solution is that of "staggering" the inlets and exits of the cross-ways so that, in effect, the conflicting traffic streams may be merged between the entrance and departure of these cross-streams, on the principles of a rotary or "traffic circle" intersection.

16.—By-pass highways should retain the traffic capacities of the main routes of which they form a part.

Costs.—

17.—The division of right-of-way costs for street thoroughfares among abutting properties, local and foreign traffic interests, the community, and other beneficiaries is so intricate a matter and especially one so affected by the peculiarities of each case, that only one conclusion concerning this division can be attempted herein, namely, that this right-of-way cost should be considered separately from the construction costs in assessing benefits and damages, or in otherwise providing funds to meet it.

18.—The cost of constructing street thoroughfares should seldom if ever be met by the abutting properties alone, but should largely, if not entirely, be provided directly or indirectly from the other-than-local traffic interests.

The minimum fraction of this cost should be borne by the abutting property when the access of this property to the thoroughfare is most restricted. The maximum fraction to be assumed by the abutting property will be when side or "service" roadways supplement the freeway of the thoroughfare so as to enable the abutting property to share fully the benefits of the latter. In this case that maximum fraction will include the costs of these service roadways

to the extent that the portion borne by others will cover the extra cost of the thoroughfare as a whole over and above that of a normal highway not a street thoroughfare.

Co-Ordination of Street Thoroughfares in the Traffic Plan.—

19.—Selection of the types of thoroughfares best fitted to serve the purposes of any particular community or area is largely affected by the topography, the character of the existing development, the existing street and road system within and contiguous to the developed area, the necessity of providing for future economies in construction of public transit facilities, and the protection of industrial as well as residential territory.

The proper inter-relation of all these factors pre-supposes a general plan covering the future metropolitan area, with legislation and resources adequate to insure its execution under continued administrative authority. Acquisition of space for future thoroughfares in advance of need, when space is attainable economically, is a primary necessity, achievable only under the authority of an official plan.

20.—Ordinarily, streets are for the use of all persons. Property on the street has the right of access; vehicles operating in the street have the right to come to the curb to pick up or to discharge passengers or goods. Vehicles using the street for purposes of public transportation, operating under a franchise, have the right to the unimpeded progress of their patrons from the sidewalk to their vehicles at points where stops are established. The pedestrian who may at one moment be crossing the street merely to get to the other side, and at another moment be boarding or leaving a public transit vehicle, has the right to complete protection in such movements. All these rights may be exercised freely, but only to the point where they interfere with the rights or safety of others. To insure that none of these rights may be exercised unduly and that the optimum welfare of all may be secured, the city may impose controls through its police power, or otherwise. It may regulate the size, weight, character, and speed of vehicular traffic; as well as the stopping of vehicles in the roadway. It may limit the movement of pedestrians or animals; and even, in the case of thoroughfares, for example, it may deny to the abutting properties access, except by special provisions of construction outside the thoroughfare roadways.

These facts have an important bearing on the design of street thoroughfares and on the distribution and co-ordination of such thoroughfares in a general city or regional plan.

Recommendation

As suggested by the foregoing principles, the Committee believes that a new type of street for automobiles is now needed in some cases, and will soon be needed generally. That type may be illustrated by the railway with its right of way closed to access from the sides, and with the important crossings, at least, not at grade.

Such a type of street would offer a solution of many of the problems now due to or connected with modern traffic, which are not solved satisfactorily by

all other means yet tried. In many cases, existing laws or customs seem to prevent the development of an ordinary highway right of way into a freeway, and that development might be impossible in any case in which the power of eminent domain of the State was not involved.

Therefore, this Committee recommends that in a city, county, or other subdivision of a State, this matter of legalizing the creation and maintenance of freeways in addition to highways, be given serious consideration, and that a study be made to the end that city and regional planners and authorities may make proper provision, when desirable, for such thoroughfares.

By the Committee on Street Thoroughfares Manual,

W. W. Crosby, *Chairman*,
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RELATION OF STREET THOROUGHFARES TO OTHER ARTERIES OF TRANSPORTATION

By Jacob L. Crane, Jr.,[6] M. Am. Soc. C. E.

The term, "artery of transportation," is graphic, but the analogy to the arteries in the human body is not quite accurate. Physiologically, an artery has the specific function of carrying the flow of blood and only that; it serves that function and only that function from the first day to the last; it does not compete with other elements of communication; and its use is automatically co-ordinated with all other organs, and the flow through it is automatically and consistently controlled. An artery of transportation, on the contrary, has none of these qualities, although it should have them, and if and when city planning finally succeeds, it will have them.

As it stands now, any one of several types of arteries of transportation may be called upon to carry the flow of any one of several types of movement; the facility which it provides is in demand by several competing elements of traffic; it is seldom satisfactorily controlled in its use and almost never properly co-ordinated with other phases of circulation and communication. As long as cities continue to grow and change in a way which it is impossible to forecast accurately in detail, the thoroughfares cannot be brought into completely satisfactory use and control and co-ordination. Meanwhile, a city's requirements must be anticipated as nearly as possible to develop the principles applicable generally and, in each case, specifically, and to improve, at least temporarily, the manner in which the traffic arteries are used, co-ordinated, and controlled. The writer is concerned with setting forth a few of the basic considerations which have to do with the inter-relation between street thoroughfares (as defined in the report of the Committee) and other principal arteries of transportation within and adjacent to urban areas. The other more important arteries of transportation may be listed as follows: (1) The roadways of secondary streets, not classed as thoroughfares; (2) the highways just outside the city; (3) footways; (4) railways; steam lines (main lines suburban lines); electric lines (main lines, interurban, suburban, street, elevated, subways, freight, and passenger); (5) waterways: rivers, canals, ports; and (6) airways.

The mere statement of these various arteries of transportation implies a definitive classification of function. The roadways of secondary streets have, as their special purpose, the carrying of wheeled vehicles for relatively short distances between points of desired access to property alongside. The outer highways are intended to collect and carry outward, and to bring in and distribute, traffic to and from the city. Footways accommodate pedestrians, bicycle riders, and roller skaters for comparatively short distances in each case. Main steam railways are designed primarily to transport passengers and

[6] Cons. Engr., Chicago, Ill.

goods between cities; suburban lines to carry them from down-town to outer residential areas; elevated lines and subways serve primarily for the movement of persons radially and transversely through a town. Waterways may have one or several functions ranging from bulk cargo barges and passenger liners to the outboard motor-boat. Thus far, airways are used almost entirely for long-distance inter-city passenger, mail, and express carriage. Some day, perhaps soon, a city plan will classify the types of movement and the arteries to carry them in even greater detail, and city planners will undertake to segregate and maintain each type of artery and each single artery for one, or at most, a few purposes.

Now, the inter-relation between street thoroughfares and these other arteries of transportation is extremely intricate. The footway, while customarily in the same strip of land with a roadway, may have a less direct relationship to that roadway than any of the other elements of transportation. As long as the footways lead to points at which the pedestrian is picked up by automobile, street car, train, or otherwise, there is no good reason why the footway needs to follow the street; in fact, there are several good reasons, safety and agreeableness being chief among them, why footways should be located quite apart from the streets.

Conversely, the roadways of secondary streets and of the outer highways must be intimately interconnected with the street thoroughfares so that traffic can flow from one to the other safely and conveniently.

While a steam railroad right of way sometimes offers a convenient location for a parallel street thoroughfare alongside, the real relationship arises out of the requirement that the thoroughfares make it possible for wheeled traffic to reach the loading points and particularly the main terminals of the railroads. The same may be said of the relationship between transit lines and street thoroughfares. On this point, however, it becomes more and more evident that no street thoroughfare (and preferably no street) will have street-car tracks, or elevated railways, or even subways, within the same right-of-way lines with the roadway. The best arrangement, and the one which may eventually be set up as a definite objective, is that in which the transit lines have rights of way altogether separate from the streets. In most cases, this would make possible the open-cut depressed transit track, bridged at street crossings and accessible from the streets at all loading points. Plainly, the practice of combining into one right of way, a street roadway, sidewalks, a surface-car line, an elevated line, and a subway, enormously decreases the efficiency of each one—least seriously, perhaps, with the subway, and most seriously with the conflict between street cars and wheeled vehicles on the pavement. The logical inter-relationship is one of providing convenience of interchange at loading points, and only that.

In relation to waterways and airways, the street thoroughfares should give direct, convenient, and fast lines of flow to the points of interchange—the passenger docks, the commercial wharves, the yachting centers, and the airports. How far they fall short of this ideal!

It is seen, then, that there are, in the main, only two desirable relationships between street thoroughfares and other arteries of transportation, namely (1) that of the flow between street thoroughfares and other streets in such a way as to cause as little interruption and hazard as possible; and (2) the relationship at points of transfer from wheeled vehicles using street thoroughfares to the car, plane, or ship using other arteries of transportation. This is simple to state, but extremely complicated in application.

Furthermore, even when a planning genius succeeds in determining a sound and appropriate pattern for all the arteries of transportation in proper relation to each other, he would still have equally difficult problems to deal with: (*a*) In the relationship between these arteries and the changing and increasingly intensive use of private property; (*b*) in the matter of creating designs satisfactory from the architectural and landscape standpoint; and (*c*) in the baffling matter of public financing.

In the planning of street thoroughfares, as in every other phase of city planning, both the big task and the big opportunity lie in devising means to bring good city building within range of the capacity to pay—within the range of financing limitations. The impassé in which the American city now finds itself (the impassé of "more to do than it can pay for"), is the result not so much of poor administration and graft, but rather of poor planning, with the necessary sequence of tremendously high costs for tearing down, rebuilding, opening up and extending the city's physical services. It seems clear enough that the "checkmated" American city can work its way out only by the most skilful, long-term planning and "programming" of its construction and reconstruction. In this project the underlying and, perhaps, the most important element is that dealing with the classification, inter-relation, and co-ordination of the arteries of transportation.

The basic principle of design for arteries of transportation is implicit in the preceding statements. Each such artery should be designed to carry specific classes of traffic to and from specific points or areas, the critical limitation on the design being the matter of cost. The several classes of traffic are passenger cars, trucks, drays, street cars, elevated cars, sub-surface cars, and pedestrians. In China, the rikshaw is an extremely important factor in street design; and, in Russia, the main boulevards must make provision for the "grand procession."

The problem of design breaks up into several elements—location, terminals, width, cross-section, grade, intersection, type and weight of surfacing, drainage, lighting, access to abutting property, and underground services. The cross-sectional design requires that the total width be built up from the items required, and there is an almost infinite variety of designs to meet special conditions. One good type is: Four 10-ft middle lanes for through traffic, two 10-ft lanes for local traffic, two 8-ft lanes against the curb for loading and unloading, and, say, a 16-ft space on each side for sidewalks, or for sidewalks and planting strip—a total of 108 ft for right-of-way width. Such a street might be classed as a street thoroughfare, but if the local traffic is

heavy, it would not operate well for through traffic. Then it would require re-study for location, or for greater width, or perhaps to separate the through lanes from the local lanes by islands, elevations, or other means.

One important consideration in design emerges from the experience of recent years, namely, that the points at which local traffic enters the thoroughfare must be infrequent, even as far apart as $\frac{1}{2}$ mile.

The principles are becoming more clear; the physical and fiscal planning is still mainly in the future.

DESIGN OF BOULEVARDS AND PARKWAYS

BY JAY DOWNER,[7] M. AM. SOC. C. E.

The design of future boulevards and parkways in the light of modern traffic conditions and the accumulated experience that is a matter of common knowledge to the Engineering Profession would appear to require added consideration to the following elements:

1.—Relationship of privately owned property to the vehicular roadway or roadways of the boulevard or parkway, with reference to the extent and proximity of linear contact.

2.—Frequency of, and the arrangement of connections between, the boulevard and parkways and the secondary or minor thoroughfares.

Boulevards long antedate parkways in origin. Behind the boulevard lies some centuries of what might be termed an evolutionary process of adaptation, modification, and improvement. Its primitive beginnings were the utilization as a public thoroughfare, of the sites of bulwarks, fortifications, or walls surrounding a town. Incidentally, the encircling boulevard in many cases happened to be a scenic highway.

By extension and analogy the boulevard became a broad, tree-planted internal thoroughfare, either as the main axis or as radials of a city plan. It provides main traffic channels and imposing settings for public buildings, grand avenues for residential uses, or for commercial and amusements enterprises, as well as its characteristically European use for the leisurely enjoyment of life in the open air during a considerable part of the year. All these purposes will be served by the boulevards of future unbuilt cities or expansions of existing cities. In width, the boulevard may range from 100 to 200 ft of public right of way utilized in varying combinations of multiple roadways separated or flanked with grass and tree-planted strips. Many examples and typical arrangements are available for study as to detailed planning.

Further evolution or modifications in the design of future boulevards would seem to indicate a need for a greater degree of segregation of the use contacts of adjacent property from the principal traffic stream and a reduction in the number of connections with lesser thoroughfares.

Parkways are of much more recent origin and were evolved from a different concept. The name itself is suggestive of what was probably the earliest conception of a parkway as a park-like thoroughfare connecting an outlying park with the down-town commercial, industrial, or built-up center of a city. Expanding combinations of this idea may cover a connecting way having park attributes and joining a park with the congested city center, or two parks with each other. More recently, it has been applied to joining many more park areas or reservations into a unified system. At the same time such an arrangement provides a traffic circulation system regardless of

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the primary purpose of connecting parks, and has brought parkways into the category of greatly extended arterial thoroughfares.

The modern parkway reservation or right of way may be, and is—in the environs of New York City., Boston, Mass., and Philadelphia, Pa.—any width from 200 to 1 200 ft. On the basis of a 40-ft paved roadway with flanking strips for seeding, planting, landscape treatment, or simply the preservation of original native beauty, the desirable minimum width is 300 ft. Intermediate widths from 400 to 800 ft may be dictated by varying topography and especially the inclusion of a stream and its immediate marginal lands. Widths up to 1 200 ft for a parkway are possible through sections of low-priced lands, such as swamps or rocky ridges.

Parkways are planned and designed on the principle of absolute exclusion of privately owned property from direct contact with the paved roadway, complete separation at all main intersecting thoroughfares of the roadway levels and their main traffic streams, and the minimum number of entrance and exit connections consistent with serving the requirements of the locality traversed. A desirable minimum spacing might be placed at ½ mile and range up to 2 miles.

Although originally they were created largely from esthetic motives, parkways have emerged as the most effective solution of the modern arterial road problem on the following purely economic grounds:

(*a*) They are efficient for passenger traffic movement because (1) they minimize interference with the traffic stream by eliminating roadside parking; (2) entrance and exit connections occur infrequently; and (3) grade crossings are generally eliminated.

(*b*) This effect on adjacent land is invariably to enhance values in contrast with a depression of values which may occur where arterial road frontage in direct contact with the pavement cannot be absorbed by business or industrial uses.

As to terminology, the Committee on the Regional Plan of New York and Its Environs, after considerable study of both legal and physical aspects, adopted the term, "boulevard," as denoting a highway that is furnished with trees, grass, or other park features, and the term, "parkway," to denote a way through, and within, a park.

These definitions are derived from the premise that, in law, there are only two classifications, namely, highways and parks. These terms have the advantage of brevity and of meeting rigid legal requirements. "Park," in a legal sense, is inclusive not only of the common conception of a park, but of a linear extension or ribbon into a parkway, even if it provides no other form of recreation than pleasure driving and enjoyment of landscape.

Although it might seem desirable to amplify the foregoing terminology as to "boulevard," the difficulty is that descriptive amplification quickly exceeds the limits of terse definition.

SUPER-HIGHWAYS

By J. P. Hallihan,[8] M. Am. Soc. C. E.

As far as is known from the records, the term, "super-highway," was coined by Daniel L. Turner, M. Am. Soc. C. E., to describe a single-level, all-purpose highway designed to provide an exclusive right of way for rail transit in combination with superior facilities for movement of heavy-volume motor traffic created in metropolitan areas.

The design was applied by Mr. Turner as the keynote of his plan for a co-ordinated thoroughfare system within the developed section of Detroit, Mich., and its expansion into a regional plan for the Metropolitan Area, later adopted as the Master Plan for Detroit and the three counties affected.

The purpose of the design may be best described by a brief condensation from Mr. Turner's report as Consulting Engineer to the Rapid Transit Commission, dated April 10, 1924:

> The specifications provide for: (1) A central reservation of 84 ft for the exclusive use of rapid transit lines on rails and the tree and shrub planting to screen it; (2) a 20-ft roadway for express motor traffic on each side of the central space; (3) a 5-ft planting or safety zone strip, on the outside of the express roadway, separating express from local roadways, but with openings at intervals for transfer from local to express where the roadways are at the same level between stations; (4) local roadways, 20 ft wide; and (5) sidewalks, 15 ft wide, making a total of 204 ft.
>
> After acquisition of the right of way, the execution of the design is intended to be coincident with the local requirements of the area and, therefore, will proceed gradually as building development progresses and traffic needs increase. At first, all roadways will be at the same level and cross-street traffic will cross the rail reservation at all street intersections. The express roadways will be provided when the central reservation is occupied by rapid transit rails and the right of way is made exclusive by separation of grades at half-mile intervals at stations, the rails and express roadways going over the half depressed cross-street and the local roadways meeting the cross-street at grade. Cross-traffic coming in between stations may then turn into the local roadway, but may cross the express roadway and rail reservation only at the under-pass. Traffic from local roadways may enter the express roadway at the barrier opening provided where local and express roadways are at the same level. Grade separations may be built for express roadways in advance of rail occupancy of the central reservation if traffic needs require, under the same limitations as to crossing.

From a traffic standpoint, the super-highway design is seen to be a provision for insurance of ample roadway space for motor traffic under conditions permitting high speed with safety. It separates directional traffic, as well as through and local traffic, and permits greater freedom of movement by reducing traffic interferences. From a transit standpoint it is seen to provide for future economies in rail transportation by making it possible to construct the permanent way on the surface at one-fifth the cost of an underground struc-

[8] Chf. Engr., Rapid Transit Comm., Detroit, Mich.

ture and at one-half the cost of a wholly elevated structure. The noise of passing trains is reduced to the minimum by the width of street and the tree screens.

A utility highway of this nature is peculiarly adapted to the metropolitan areas surrounding great cities, where traffic volume assumes great proportions and where mass transportation facilities must eventually provide for a density beyond the capacity of a street car and bus system. Beyond those areas the safety provisions of separation of directional traffic and permissible elimination of left turns at intersections may be maintained by a boulevard design with a minimum of 30 ft for the central parkway, and with side parkways separating distance traffic from local traffic.

There is no economic reason why the all-purpose roads of the future should not provide for esthetic treatment by landscaping as well as for the utility needs, provided an adequate right of way is secured upon initial location, before it acquires a prohibitive value created by the facilities furnished.

THOROUGHFARE LAYOUTS IN RELATION TO DRAINAGE SYSTEMS

By W. W. Horner,[9] M. Am. Soc. C. E.

For the purpose of this paper, the "thoroughfare" is the general designation of a traffic-way of importance and will include streets or roads generally called "boulevards," "arterial highways," "major or secondary highways," and "parkways." The "drainage system" of the region comprises all facilities for the removal of storm water, ground-water, spring flow, and other excess liquids, exclusive of sewage and contaminated waste waters. These facilities may be the natural watercourses, such as creeks, brooks, dry runs, or ravines, or may be artificial drains, either open or covered.

The construction of covered masonry storm drains of large size, as is now common in large cities, is:

(1) The result of failure to develop and organize a proper system for the improvement and protection of natural watercourses. This failure is largely due to improper platting, which either disregards watercourses or leaves the valleys as waste land, or places the watercourses at the rear of lots, where they readily become rubbish dumps.

(2) Placing a high charge on platted property for which there is no adequate return and which is accordingly detrimental to simple and uniform development and tends to produce high density of population.

(3) Destructive of the greatest asset for a residential setting, an attractive natural scenery of a semi-pastoral character.

The alternative drainage policy involves the preservation of watercourses enlarged or supplemented to become adequate for the increased run-off from urban areas and the protection and preservation of the natural conditions. tional purposes;

(*a*) The setting aside of considerable areas of lowlands for permanent use as drainage ways, but with possibilities of considerable value also for recreational purposes:

(*b*) The subdivision of land so that drainage ways will be conspicuously evident, in order that their scenic value will be best appreciated and that they may not be readily restricted by filling or surreptitiously used for the disposal of rubbish; and

(*c*) A consequent requirement for lot frontage along the drainage ways which would then involve the platting of marginal streets along such ways. The valleys being generally easy gradient routes, such streets would naturally be thoroughfares in the direction of the valley trend.

General

Proper original planning should provide a system of thoroughfares bordering reserved natural drainage ways to which would be joined a complementary

[9] Cons. Engr., St. Louis, Mo.

system crossing the valleys at the most advantageous points. This system has been followed for residential areas and has resulted in such major residential streets as those of the Turtle Creek Valley, in Dallas, Tex.; such boulevards as those of Kansas City, Mo.; and such parkway developments as in Westchester County, New York State. Common habit to the contrary, the system can be used also in commercial areas if people are willing to organize adequate protection and policing. The result would be to the ultimate benefit of the community from every angle. There is a popular reaction against the present ugly, uniform type of urban development which would welcome a change in this direction. Obviously, the scheme is not generally applicable to thoroughfare planning of a corrective character, in old settled communities, and it has no economic advantage where money has already been spent for large storm sewers.

EXPRESS HIGHWAYS AND BY-PASS ROUTES

By Daniel L. Turner,[10] M. Am. Soc. C. E.

Express highways and by-pass routes are recent evolutions of the motor-vehicle age. There was no necessity for them in a man-and-horse civilization. By-passes have been in use for a considerable time, but express highways have developed very slowly. The latter came into actual use only a short time ago. This, too, despite the fact that H. G. Wells anticipated such special types of roads for motor-vehicle use as early as 1901, and at the very beginning of the motor era, when he described the probable "Locomotion in the Twentieth Century," as follows:

"The * * * conceivable ideal of locomotive convenience, * * *, is surely a highly mobile conveyance capable of travelling easily and swiftly to any desired point, traversing, at a reasonable controlled pace, the ordinary roads and streets, and having access for higher rates of speed and long-distance travelling to specialized ways restricted to swift traffic * * *. They (the specialized ways) will have to be very wide,— * * * just as wide as the courage of their promoters goes—and if the first are too narrow, there will be no question of gauge to limit the later ones. Their traffic in opposite directions will probably be strictly separated * * * and streams of traffic will not cross at a level * * *."

At another point, describing the use of the privately owned motor carriage on such special ways, he stated:

"This, for all except the longest journeys, will add a fine sense of personal independence. * * * It will be capable of a day's journey of three hundred miles or more, * * *. One will be free to dine where one chooses, hurry when one chooses, travel asleep or awake, stop and pick flowers, * * * be free to travel up to the limit of their very highest possible speed."

He thought such segregation of motor traffic might possibly begin even in the decade from 1900 to 1910. People are just now (1934) beginning to think about it seriously. The foregoing is a remarkable vision of the conditions of to-day at a time when the tremendous growth of automobile use scarcely seemed conceivable.

The express feature of a highway applies only to the roadway as it is defined in the definitions by the Committee on Street Thoroughfares. Therefore, an express highway may be defined as a highway having an express roadway. On the same basis, there may be express boulevards, express parkways, express streets, express roads, etc. In fact, every type of highway would become express if provided with an express roadway.

An express roadway may be defined as a roadway constructed with separated opposite-direction lanes and without crossings at grade, so as to permit fast minimum-stop traffic.

The ideal express roadway should not be directly accessible to abutting property; and access to it from the local lanes of the highway of which it may be a part should be infrequent. Similarly, it should be accessible only infrequently to other highways. There should be lanes for trucks and buses as well as passenger cars. Grades should be low. Curvature should be reduced

[10] Cons. Engr., New York, N. Y.

to the minimum. There should be at least two traffic lanes in each direction and, preferably, three.

Practical considerations will make necessary many departures from the ideal. Other types of highways have been, and will be, proposed to secure to as great a degree as possible the advantages of the ideal express roadway—without resort to the high construction costs and to such complete differentiation as the ideal requires.

Instead of including the express roadway as a feature of the highway it may be constructed as an entirely separate highway without any local traffic lanes, but designed for express traffic exclusively. In order to distinguish it from an express road or express street which provided for both local and express traffic, such a highway might be called an expressway; that is, defined as a highway for express traffic exclusively. This would be the ideal method for express service.

Along such expressways, at signal-controlled ingress and egress points, all the necessary service facilities should be segregated, restaurants, inns, or hotels, comfort and rest houses, oil and gas, repairs, emergency trucks, etc. These service stations should be as far apart as practicable, consistent with the development of the country side. Along the expressway, however, emergency pull-out spaces, accessible only from the roadway, should be provided at convenient distances apart and these should be equipped with telephone to the service stations. Here travelers could change tires and make minor repairs without resort to the station service, and without obstructing the traffic lanes.

Expressways will not be as practicable of construction in urban districts as in the open country. An elevated or depressed roadway in a city makes an ordinary street into an express street, it is not an expressway in the sense intended herein.

The most important city streets should be those planned for the accommodation of mass passenger transportation facilities on rails, as well as on rubber tires because, after all, probably three-fourths of the people travel along such thoroughfares. In fact, such streets, which may be termed super-highways, should be the base plan for any city street system. After they are planned to circulate and distribute the population in the best possible manner, all other streets, which are secondary, should fill in and complete the circulating system. Failure to recognize the necessity of providing ways for these mass travelers is one of the greatest oversights of which city planners have been guilty in the past. The present congested condition of cities is largely due to this oversight.

A super-highway may be defined as a highway designed to accommodate mass passenger transportation facilities on rails and rubber tires, in addition to all other street uses.

Since a by-pass route is only a connecting link around a locality between highways into, and out of, the locality, it should conform in all particulars to the type of the highways it is to connect permanently; that is, the by-pass should be of the expressway type if eventually it is to form a part of the expressway or, if it is to shunt an ordinary road around congestion, it should be an ordinary road.

PAYING FOR STREET THOROUGHFARES

By George H. Herrold,[11] M. Am. Soc. C. E.

Historical

In 1785 the Continental Congress established the township system originated in New England, and made it applicable to all Government-owned land. The State then adopted or created the section-line highway of 2, 3, or 4 rods in width, to make the lands accessible and to permit their development. This was practically the highway system of the United States—a rectangular system. The easement to the road being by prescription, the abutting owners, in turn, had access to the road at all points of contact for trade and barter of the products of the land. Originally, the township paid for the construction of the roads on the section and quarter-section lines; then, some one in New Jersey applied the "scientific method." By observation on a certain stretch of road, he found not only local township wagons, but wagons from adjoining townships and from townships far beyond. He reached the conclusion and was able to convince the Legislature that the State should contribute. The first legislative act was to require that the abutting property owner pay one-tenth of the cost, the State one-third, and the county the remainder. Thus began, in principle, the taxing of urban units for the development of country roads.

Toll Roads.—The first attempt at creating thoroughfares of communication was the establishment of toll roads in the Eighteenth Century. The first cost and maintenance was borne by chartered turnpike corporations, and the users paid a toll to reimburse the owners. These companies had the right of eminent domain and could appropriate existing highways when necessary. The turnpikes were public highways, in every respect, and were free to all except that the legal toll must be paid as a condition of use.

One of the great toll roads of history was constructed by the United States Government. The Cumberland Road, or National Turnpike, from Fort Cumberland (Baltimore, Md.), across Pennsylvania, Ohio, and Indiana, to Vandalia, Ill., about 800 miles, was a Government project. It was begun in 1806 and finished in 1844 under a Congressional Act of 1796. It was financed by the United States Treasury which was supposed to be reimbursed from a road fund. This road fund was created by setting aside one-tenth of the proceeds of the sale of Government lands in Ohio and possibly other States. The cost of the pike was $7 000 000, and it carried an annual traffic of 3 000 wagons.

Government aid for road building was stopped about 1840. It was then advocated that the States should build their own roads and the Government confine its financial aid to territorial and military roads.

Steam Roads.—Coincident with the development of the toll road, but beginning later, was the wonderful and rapid promotion of the steam rail-

[11] Member, State Planning Board; Managing Director and Engr., The City Planning Board, St. Paul, Minn.

road. By 1840, it had demonstrated its ability to compete with toll roads. It operated over a private right of way and its destination was a succession of terminals. Owners of abutting property had no right of access to its right of way except at terminals. Encroachment on the right of way or trespass was punishable by law. Land was often given for terminals by abutting owners who recouped by enhancement in value of their remaining land; or the railroad companies bought the land at a proposed terminal and reaped the benefit of this increase in value—a benefit brought to it by the railroad which made it accessible to other terminals.

Railroads were financed by private capital, land grants, gifts, and Government loans. Shippers paid the entire cost of moving their products or commodities over the railroads.

Trunk Highways.—The present-day road-building activities began with Federal aid. For twelve years, appropriation bills were written and killed prior to 1916, when the Act was successful. It provided that the Federal Government should extend aid to the States for the construction of rural post roads and for other purposes. These enormous expenditures ($125 000 000 per yr in 1931, 1932 and 1933, of Government aid matched by an equal amount from the States) made possible the great network of interconnecting trunk highways over the United States.

State funds are produced from gasoline tax, motor-vehicle license tax, wheelage tax, mileage tax, and bonds.

Urban Streets.—In urban centers the principal use of land is for streets, parks, and building sites. The street system is usually created by dedication in a town-site plat. Many cities were started during a great speculative era when more thought was given to land as a commodity and the profits to be made from it than the development of a street system for a city.

Widths of streets were determined by the vision, generosity, or cupidity of the owner or developer. In fact, so little was known about street widths that habit probably determined many of them, and the 60-ft street became fairly common. It was considered a suitable boundary for lots.

In some notable instances, thoroughfares of ample width were laid out to serve the street system of the business center, but more often no thoroughfares were contemplated. For speculative reasons certain uses were established on these streets. Some of these uses could have been better served on wider streets while others could be served on narrower ones.

From 1870 to 1920 city populations 'worried along" with this inadequate and poorly designed street system, and improper location of structures and uses.

The introduction of the motor vehicle has compelled a larger floor area for transportation uses. Thoroughfares have had to be created to serve centers and minor street systems. There has been much widening of mercantile streets and connections and construction of parkways for passenger cars, arterial streets, express highways or super-highways—all-purpose thoroughfares and cul-de-sac street systems in residential districts. Where not created in new land these have been obtained at an enormous cost.

Financing Public Improvements

The procedure for financing public improvements varies from place to place and from State to State; but, speaking generally, when a group of citizens want their street graded, widened, or paved, or when they want a sewer put in or a sidewalk built, they enter into a co-operative agreement with the municipality, under which the latter does the work and those who benefit by the construction pay for it. The State gives the right to a governmental unit to do this under quite elaborate legal protective restrictions.

The necessity of a municipality handling these co-operative individual investments in local improvements can be explained by one simple example. If a group of citizens wishes to construct a sidewalk the following are some of the questions that would arise. To what grade should the walk be laid? (Each owner will want the walk as near to the level of his property as possible.) Shall the walk be 4 ft or 6 ft, or some other width? Shall it be built of concrete, brick, or cement tile? Shall it be a cinder or board walk? Shall it be laid along the property line or in the middle of the reservation, or along the curb? Is a walk necessary at all? Who would underwrite the construction cost in order that a contract might be let? How would collection of each one's portion of cost be made?

It is not possible for a group of people to arrive at an agreement on these things, due to difference in culture, experience, knowledge, notions, and financial ability to pay, and if each laid his own walk a construction Babel would be the result. The municipality, therefore, carries out the work according to its engineering standards; pays the contractor from moneys which it has arranged beforehand to have on hand through a tax levy or bond issue or certificates of indebtedness, and collects back from the citizens the cost of the work, justly apportioned to each according to benefits, in installments with interest. If these costs are not paid they become a lien on the property and are added to the taxes, and when the taxes are paid the assessment must be paid at the same time, or the property is sold.

In many States a municipality also has the power to order such improvements as a public convenience and necessity and to collect the cost from owners benefited.

The three great powers of Government are: The power of eminent domain; the power of levying taxes; and the power of collecting the construction cost for local improvement work from property owners as a benefit assessment. No greater power exists. Ownership of land infers simply the right of possession contingent on payment of taxes and assessments; otherwise, possession passes to others.

In street thoroughfare widenings, the municipality takes the property required under its powers of eminent domain, orders buildings moved or cut off, resets the curb, widens the pavement, constructs sidewalks, changes grades, moves light standards, fixtures, etc., all under its power of investing the citizen's money in street improvements to enchance his business and social opportunities. It pays the owner for the land and building taken, the cost of new front remodeling, consequential damages due to the construction work,

etc., and it pays the contractor. It then proceeds to collect the cost from citizens that are benefited. This may be done by a special assessment or through a tax levy, or both.

If it can be shown that certain property receives a benefit or peculiar advantage by construction work authorized by the municipality, that property may be assessed for the cost. It is the property owner's investment in an improvement. If the benefit or advantage is to the entire city, every piece of property in the city may be assessed. This is probably never done, as it is simpler to spread the cost on the tax roll and collect it with the real estate tax.

If the benefit or advantage is greatest to abutting property on a street, but also extends to a district or to the entire city, but in a lesser degree, a graduated assessment may be spread over the street or district, and the remainder may be collected as a tax on real estate, remembering that the ones assessed must also pay the tax, because a real estate tax covers every parcel. If the benefit or advantage is entirely to the street the abutting property on that street may be assessed for all the cost.

A benefit assessment is a bill for construction work done by the municipality for a group of property owners to enhance the social, economic, and physical usefulness of their property. To insure uniformity of construction, compliance with engineering specifications, and equitable distribution of the cost, the municipality makes the investment for the property owners and collects the bill.

To determine the value of thoroughfare improvement is at best a matter of forming an approximate estimate.

In Providence, R. I., all paving is paid for out of a street-paving fund set up in the budget each year on the theory that all citizens were benefited by paving. Cities and towns in Virginia are constitutionally prevented from levying special assessments for local improvements, except that cities of more than 500 inhabitants per sq mile may assess abutting property owners for sidewalks in existing streets, for improving and paving existing alleys, for constructing sewers, or for the use of sewers.

In St. Paul, Minn., it is assumed that 24 ft of paved roadway is sufficient to serve abutting property and that if a greater width is necessary it is for the benefit of the city at large. It is a charter provision that 24 ft, or less, of pavement may be the investment of the abutting property and that such an investment cannot be required of the owner oftener than once in fifteen years. The remainder of the cost is the city's investment and amounts to from 44 to 58% of the total cost. The amount to be assessed for a sewer is decided for each particular case; there is no limitation, the amount assessed depending on the value of the property and the benefits. On the other hand, water mains are assessed at $1.00 per front ft, payable in ten yearly installments.

In Minneapolis, Minn., one-third the cost of paving is paid for by abutting property, one-third from the current paving fund, and one-third from bonds running for thirty years. Abutting property owners may be assessed $2.50, or

less, per front ft for a sewer, the remainder being paid from a bond issue or by general taxation.

These are arbitrary rules laid down for convenience and to prevent difficulties that would result from more refined methods. Sometimes they are inconsistent, but they work. The methods used in each city are usually considered, if not absolutely equitable, at least the best they can "get away with."

In street widening, or openings, the apportionment of cost is not so simple—land must be acquired and owners want money for land. No simple rules have yet been devised such as are used for other public improvements. The owner demands all he can get in damages and escapes all he can in the award of benefits. Land values are determined by appraisers; the building damages are determined by engineers, architects, or contractors.

The building damages can be determined accurately and beyond serious dispute, but the determination of land values is difficult. Expert testimony may develop differences that cannot be reconciled. Such expert testimony reflects the psychological attitude of the individual. It may be based on facts, or it may reflect an intoxication of optimism and imagination. The municipality must accept a reconciliation of these mental attitudes as representing the actual value and may proceed to pay the cost and to apportion the benefits to the property which, in its judgment, has received an advantage or benefit.

The apportionment of the amounts of assessment in each case is the most intricate problem confronting a city.

In 1913, the City of St. Paul widened Robert Street, a mercantile street. After considering various ways of financing the project, the majority of abutting owners on the street agreed with the City that they would pay the cost. The City instituted condemnation proceedings acquiring 20 ft on the west side from Second Street to Central Avenue, twelve blocks, of which six blocks were in the heart of the retail district. The street was widened from 55 to 75 ft. Theoretically, the owners on the east side of the street who were undisturbed paid one-half the value of the land and buildings taken on the west side. The owners of the property on the west side, which was cut off from the widening, accepted one-half the value of what they gave up. The entire street was repaved, and both sides paid. The cost was $1 000 000, or at the rate of $250 360 per mile.

In 1923, the widening of Robert Street was extended north 1 500 ft., connecting with University Avenue by cutting through a hill and separating the grade with one of the cross-streets (Cedar Street). This made a continuous thoroughfare from the University of Minnesota, in Minneapolis, to the heart of the business district of St. Paul, a distance of six miles. This extension cost $260 000, and it was assessed largely over that part of Robert Street originally assessed. It was regarded as a completion of that project.

In 1927, the new Robert Street Bridge with a 56-ft roadway was built over the Mississippi River, at a cost of $1 800 000, and through a period of three years other widenings were completed, making Robert Street a con-

tinuous thoroughfare to South St. Paul, three miles, and there connecting with a State trunk highway.

The value of property on Robert Street was never enhanced to equal the assessment of $1 260 000, although it was approaching it in 1928 when the value curve turned downward. The street was widened to preserve its business character, to make the business houses more accessible, and to prevent mutation. It was the desire of the owners in 1913 to make it an arterial street and this was accomplished.

On Robert Street, at Seventh, are two large department stores, each of which do an annual dollar volume business exceeding that of any other retail store in the Northwest. There are also two large clothing houses, banks, hotels, office building, brokerage houses, restaurant, and shops. The widening of the street started values upward. The increase of through motor travel pulled values downward.

Seventh Street, in St. Paul, runs at right angles to Robert Street. It is the leading retail street and on it may be found the highest land values in the retail district. It is a 60-ft street with a 40-ft roadway, and 10-ft sidewalks; and it accommodates a double-track car line. The Planning Board recommended that the street be arcaded for five blocks, placing the sidewalks within the building and widening the roadway to 56 ft. During the discussion, property owners asked for an estimate of cost for a physical widening, making an 86-ft street with a 56-ft roadway, and two 15-ft sidewalks. The estimated cost was approximately $6 500 000, but an analysis of these figures disclosed that if the property owners would give the land and each pay his actual reconstruction cost; that is, tearing down 13 ft of his building, constructing a new front, re-arranging the interior, paving an additional 8 ft, and building a 15-ft sidewalk, the actual cash outlay on the part of the property owners would have totaled approximately $1 000 000. This difference is the so-called value of the land and buildings taken. Now, if the City had proceeded with condemnation proceedings and allowed damages amounting to $6 500 000, and then declared benefits of $5 500 000, the same results would have been obtained, but the property owners and every one else knows that such an award of benefits would be attacked, although it is equitable and just, and the property owners would stand a good chance of winning their case in Court, because of the attitude of the Courts toward property values.

Where street widenings are necessary, it is because the original dedication of width was not sufficient. The original dedication for a street was made from the land, the owner absorbing his loss of area and benefiting by accessibility to all parts of the remaining portion. Where a street is widened by the power of eminent domain, it is to correct the error of the original dedication.

The L'Enfant Plan of Washington, D. C., provided 90, 110, and 160-ft streets. These wide streets were no burden to the city or to the property owners, as only that part was improved that was needed for traffic—the abutting owners using the excess width of street as yards. When it is necessary to widen a street in Washington, no condemnation proceedings are necessary, the roadway is widened by setting back curbs and sidewalks.

Users Should Pay

The widening of a street can only be supported by arguments based on surveys and a careful analysis of the survey data. Physical units are not the only ones to be counted. There is a social aspect, an economic condition, and a psychological viewpoint. Furthermore, if there is a chance of decreasing the cost of distribution, the advantage accrues to the owner, to the traffic, and to the customer.

In a city the character of use determines the width of the street and the width of the roadway, the latter being based on the almost uniform laws establishing vehicle widths. The following road width (in feet), will be considered standard:

For residence districts, minor streets	27
For residence districts, major streets	36
For approaches to residence districts, or arterial streets	36
For mercantile businesses, or for multiple-family homes	56

Arguments for these roadways may be found in all literature on the subject. They are the required roadway widths for such uses of property. The width of the street is the width of the roadway required, plus the width of the sidewalk required, plus planting and embellishment strips desired. The owners of abutting property should pay the entire cost of the right of way or street width in these cases.

The paving or surfacing of these roadways and the question of who shall pay the cost must be determined locally, depending on values, on use, and on general custom. There is always some through traffic on all streets in a varying volume. If owners of motor vehicles pay a personal property tax, a gas tax, and a wheelage tax (or some combination of the three), these moneys should be set aside for paving or surfacing purposes.

For arterial streets, express highways, super-highways, and trucking ways, consideration must be given to acquiring the land for a right of way or for a widened right of way through general taxation of, bonds of, and special assessments of, districts or terminals. It would be unusual for the owners of abutting lands to profit; the property may even be depreciated by these thoroughfares unless a buffer right of way is provided, broad enough to protect the abutting owner from the noise, heat, and fumes of traffic.

The right of way for a parkway should be wide not only for landscape purposes, but (more important) for protecting the near-by property owner from the undersirable features of traffic; and, where there is no access of abutting property, there should be no assessment except where the property may be a part of the assessment district. If service drives for abutting property are constructed on the parkway lands, the value of this part of the land and the construction cost should be assessed to the abutting property, providing the development has not taken away an existing frontage right, in which case the law would require that the service roadway be provided at the cost of the municipality building the parkway. In general, there is a right of access to all property which should be paid for by the owners of that

property, but the right of access may not be by way of a parkway, artery, or super-highways, except as use is made of the same right of way.

In the case of highways between terminals the same principles apply. The right of way must be acquired and paid for by assessments of the district, or preferably the terminals, and the grading, paving, and structures by the users —motor-operated vehicles through license fees, gas tax, and wheelage tax.

It would seem to be a basic principle that land for streets or for street widenings, of the standard widths required by the use, should be provided by abutting property without cost; that is, in condemnation proceedings, benefits and damages should be made equal for land acquirements. Why should the owner be paid for the increment in value that has been created by the people of the city? The only difference in the value of a lot in the Sahara Desert and a lot on Fifth Avenue, in New York City, is the spirit of business that surrounds that lot, and that spirit of business was created by the people of New York. The owner receives his remuneration in the increased value of his remaining property and in obtaining the accessibility which his business required.

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AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

ECCENTRIC RIVETED CONNECTIONS

By EUGENE A. DUBIN,[1] Esq.

Synopsis

Formulas and corresponding curves for designing riveted connections, subjected to eccentric loads, are developed in this paper. The general equations express, directly, the relationship between the load, the eccentricity of the load, the stress in the extreme rivet, the spacing of the rivets, and the number of rivets in the group.

Although riveted connections subjected to eccentric loads are frequently encountered in practice, the method of designing them is essentially a matter of "cut and try"; that is, the rivet group is assumed and the stress in the extreme rivet is checked. Hitherto, "families" of curves have been made for use in designing this type of connection, but it has been difficult to interpolate for values between such curves, and beyond the range of the curves the "cut-and-try" method must still be used. Furthermore, the probability of reading values, incorrectly, is great, due to the complicated nature of such curves. Tables are also available, but they, too, offer no accurate means of interpolation for values other than those listed.

The formulas and charts presented with this paper are suggested to replace those now in current use. Accurate and quick results are possible with these curves due to the inherent advantages of an alignment chart over a "family" of curves. The general formulas herein developed, express directly the relationship between the load, the eccentricity of the load, the stress in the extreme rivet, the spacing of the rivets, and the number of rivets in the group; but the resulting function of n (the number of rivets), is difficult of solution algebraically. However, a graphical solution of this function is possible by means of an alignment chart. The problem has been restricted to a consideration of the case in which the action line of the load is parallel to the rows of rivets in the group. Figs. 1 to 4 are offered for the solution of cases that occur most commonly. The charts are for groups of rivets containing from one to four rows, and for the one case of 3-in. spacing of rivets. Fig. 2 is made to cover all cases in which the spacing between rows of rivets varies from 3 to 10 in. For connections of three and four rows of rivets only one case is presented—that of 3-in. spacing between rows. For cases beyond the

Note.—Discussion on this paper will be closed in November, 1934, *Proceedings*.

[1] Washington, D. C.

range of the charts, in which the "cut-and-try" method is used, the formulas for the polar moment of inertia of the rivet group, and the distance from the center of gravity of the group to the extreme rivet will aid materially in the solution.

For checking the strength of connections not covered by the charts the formulas developed are quick and easy of solution. Fig. 1 is useful for designing plate-girder web splices when the common assumption is made that the stress in each rivet, due to bending, is in direct proportion to its distance from the neutral axis of the group. To design web splices, then, it is only necessary to divide the shear and moment of the web uniformly between the assumed number of rows of rivets on each side of the splice, determine the eccentricity of the shear (the moment divided by the shear), and proceed to use the charts as for any eccentric load. In the same manner, Fig. 1 may be used for a preliminary design for connections of more than one row of rivets that lie beyond the range of the other charts.

Formulas and Derivations

To use the charts, lay a straight-edge across the three scales of the chart, intersecting two of them at known values. The point at which the straight-edge intersects the third scale is the desired value. The following notation is presented for the convenience of discussers:

a = a number; thus: a $(a + 1)$, $(2a + 1)$, etc.;
d = diameter of rivet;
e = eccentricity of load with respect to the center of gravity of the rivet group;
n = total number of rivets in a group;
p = pitch;
r = distance from the center of gravity of the group of rivets; r_0, to the extreme rivet; r_1, r_2, etc., to Rivets 1, 2, etc., of a group; and Σr^2 = polar moment of a group;
s = total stress in extreme rivet, or allowable total stress;
w = width of a strip, or distance between rows of rivets parallel to the axis of the member;
x = an unknown variable;
$A = \frac{P}{s}$
P = load, with line of action parallel to the rivet rows;
S = section modulus of a group of rivets;
θ = an angle between vectors (Fig. 2(*d*));
Σ = summation; Σr^2 = polar moment of inertia of a group of rivets; and,
ϕ = an angle measured at the center of gravity of a rivet group (see Fig. 2(*d*)).

Example 1.—One Row of Rivets.—The stress in the extreme rivet (Fig. 1(*a*)) is the resultant of the shearing stress and the stress due to bending, which, for one row of rivets, is at right angles. Therefore, $s^2 = \left(\frac{P}{n}\right)^2 + \left(\frac{P\,e\,r_0}{\Sigma r^2}\right)^2$; or,

$$\left(\frac{s}{P}\right)^2 = \left(\frac{1}{n}\right)^2 + \left(\frac{e\,r_0}{\Sigma r^2}\right)^2 \qquad (1)$$

in which,

$$r_0 = p\,\frac{(n-1)}{2} \quad \text{.........................(2)}$$

and $\Sigma\, r^2 = r^2_1 + r^2_2 + \ldots + r^2_0$. With an odd number of rivets in the row: $r_1 = p$, $r_2 = 2p$, etc.; and,

$$\Sigma\, r^2 = 2 \sum_{1}^{0.5\,(n-1)} (p\,x)^2 = 2\,p^2 \sum_{1}^{0.5\,(n-1)} x^2$$

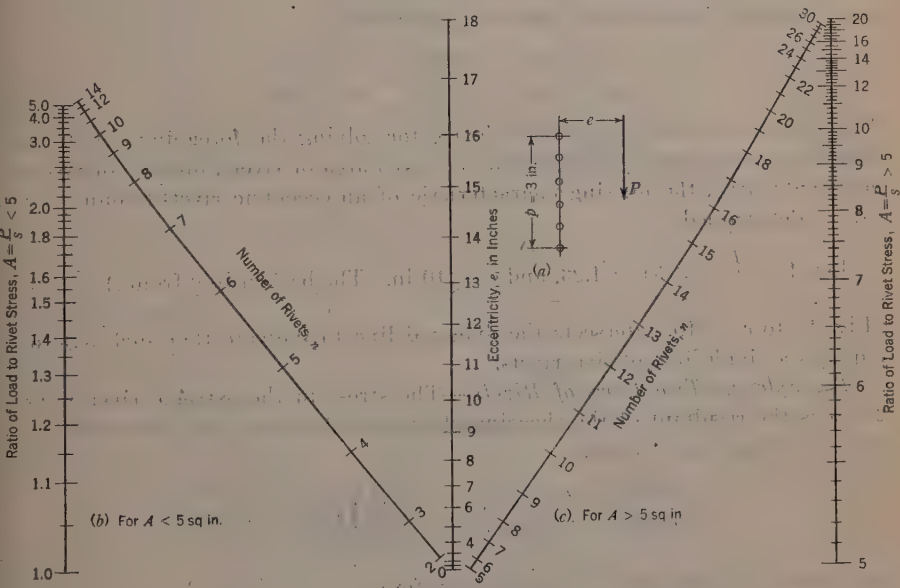

FIG. 1.—ONE ROW OF RIVETS WITH A 3-INCH PITCH.

The sum of a series in x^2 between the limits, 1 and a, is given by the expression, $\sum_{1}^{a} x^2 = \frac{a\,(a+1)\,(2\,a+1)}{6}$. Therefore,

$$\Sigma\, r^2 = \frac{2\,p^2}{6}\left[\left(\frac{n-1}{2}\right)\left(\frac{n-1}{2}+1\right)(n-1+1)\right]$$

$$= \frac{p^2\,n}{12}\,(n-1)\,(n+1) \quad \text{..................(3)}$$

Although the expression for the polar moment of the rivet group (Equation (3)) was derived for a row with an odd number of rivets, it can be proved true for a row with an even number of rivets.

Equations (2) and (3) may be substituted in Equation (1) and the result written as,

$$\left(\frac{1}{A}\right)^2 - e^2\left[\frac{6}{p\,n\,(n+1)}\right]^2 = \frac{1}{n^2} \quad \text{.................(4)}$$

in which, $A = \frac{P}{s}$. For constant values of p this equation takes the form,

$$f_1(A) + f_2(e) \times f_3(n) = f_4(n) \qquad (5)$$

a standard type of equation soluble by an alignment chart.

The section modulus, S, of any group of rivets is:

$$S = \frac{\Sigma r^2}{r_0} = \frac{p\,n}{6}(n+1) \qquad (6)$$

and, therefore, Equation (4) may also be written:

$$\frac{1}{A^2} = \frac{1}{n^2} + \frac{e^2 r^2_0}{(\Sigma r^2)^2} = \frac{1}{n^2} + \frac{e^2}{S^2} \qquad (7)$$

The alignment chart, Fig. 1, is a device for solving the foregoing equations. With any two of three values known (required area of rivets, number of rivets, or eccentricity), the missing characteristic of an eccentric riveted connection can be determined.

Let $A = \frac{P}{s}$ $(< 5) = 1.23$, and $e = 10$ in. The broken line from $A = 1.23$ (Fig. 1) to $e = 10$, intersects the diagonal line to indicate that such a joint requires a single row of five rivets.

Example 2.—Two Rows of Rivets.—The stress in the extreme rivet (Fig. 2(a)) is the resultant of the shearing stress and the stress due to bending, as shown in Fig. 2(d). Then,

$$s^2 = \left(\frac{P}{n}\right)^2 + \left(\frac{P\,e\,r_0}{\Sigma r^2}\right)^2 - 2\left(\frac{P}{n}\right)\left(\frac{P\,e\,r_0}{\Sigma r^2}\right)\cos\theta$$

and, $\cos\theta = -\cos\phi = -\frac{w}{2\,r_0}$. Therefore,

$$s^2 = \left(\frac{P}{n}\right)^2 + \left(\frac{P\,e\,r_0}{\Sigma r^2}\right)^2 + \frac{P^2\,e\,w}{n\,\Sigma r^2}$$

or,

$$\left(\frac{s}{P}\right)^2 = \left(\frac{1}{n}\right)^2 + \left(\frac{e\,r_0}{\Sigma r^2}\right)^2 + \frac{e\,w}{n\,\Sigma r^2} \qquad (8)$$

The vertical component of r_0 may be obtained from Equation (2), and the horizontal component added to it vectorially; thus

$$r_0 = \left[p^2\left(\frac{\frac{n}{2}-1}{2}\right)^2 + \frac{w^2}{4}\right]^{\frac{1}{2}} = \left[\frac{p^2}{16}(n-2)^2 + \frac{w^2}{4}\right]^{\frac{1}{2}} \qquad (9)$$

The vertical components of Σr^2 for one row of rivets are obtained from Equation (3), with n replaced by O. 5 n, and the result is multiplied by two for the two rows of rivets. The horizontal components of Σr^2 are $n\left(\frac{w}{2}\right)^2$;

therefore,

$$\Sigma r^2 = \frac{2p^2}{12}\left(\frac{n}{2}\right)\left(\frac{n}{2}-1\right)\left(\frac{n}{2}+1\right)+\frac{n w^2}{4}$$

$$= \frac{p^2 n}{48}(n-2)(n+2)+\frac{n w^2}{4} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

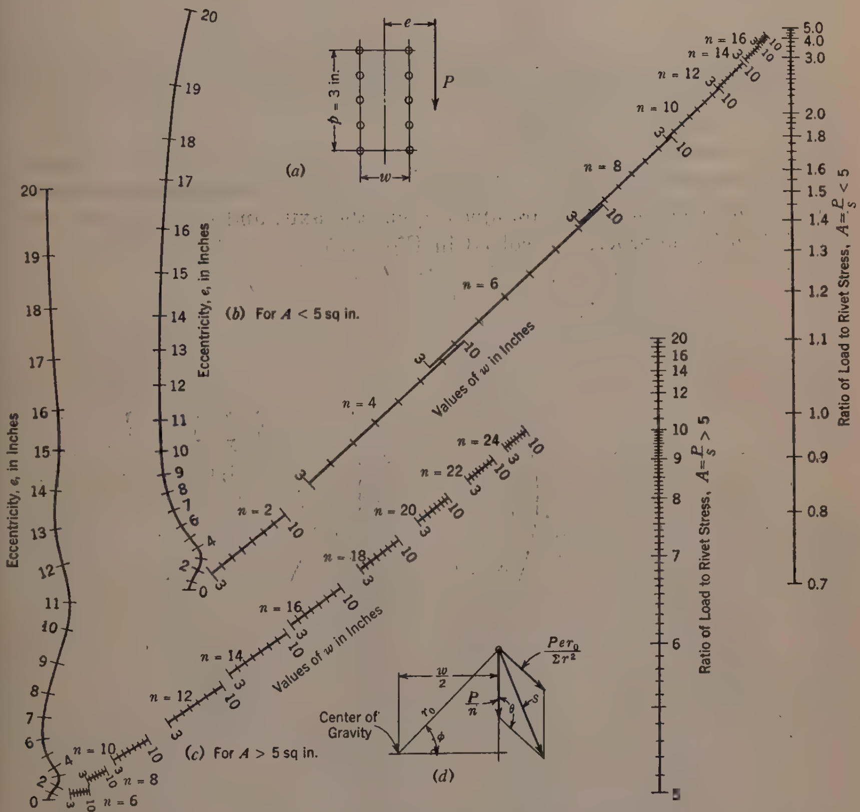

FIG. 2.—TWO ROWS OF RIVETS WITH A 3-INCH PITCH.

Equations (9) and (10) may be substituted in Equation (8), but the resulting function cannot be transformed to fit any standard form for which an alignment chart may be plotted. Nevertheless, a graphical solution is possible by a method developed by Mr. A. Wertheimer, of the Bureau of Ordnance, United States Navy Department. The result may be brought to any desired degree of accuracy by successive approximations. The charts for two, three, and four rows of rivets were constructed by this method. In no case will the error in the charts exceed 5 per cent.

Referring to Fig. 2, a formula similar to Equation (7) may be written:

$$\frac{1}{A^2} = \frac{1}{n^2} + \frac{e^2 r_o^2}{(\Sigma r^2)^2} + \frac{e w}{n \Sigma r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

For this case,

$$r_o = \left[\frac{p^2}{16}(n-2)^2 + \frac{w^2}{4}\right]^{\frac{1}{2}} \qquad (12)$$

and,

$$\Sigma r^2 = \frac{p^2 n}{48}(n-2)(n+2) + \frac{n w^2}{4} \qquad (13)$$

The n-curves of Fig. 2 are a series of curves for constant values of n and varying values of w, in which, w varies from 3 in. to 10 in. The n-curves are graduated in units of $w = 1$ in.

Fig. 2 is used precisely the same as Fig. 1. With the numerical data in Example 1 enter Fig. 2(b) as shown by the broken line, which indicates a joint of four rivets, about $8\frac{1}{4}$ in. apart along the axis, and a 3-in. pitch. If A is more than 5, the problem is solved in Fig. 2(c).

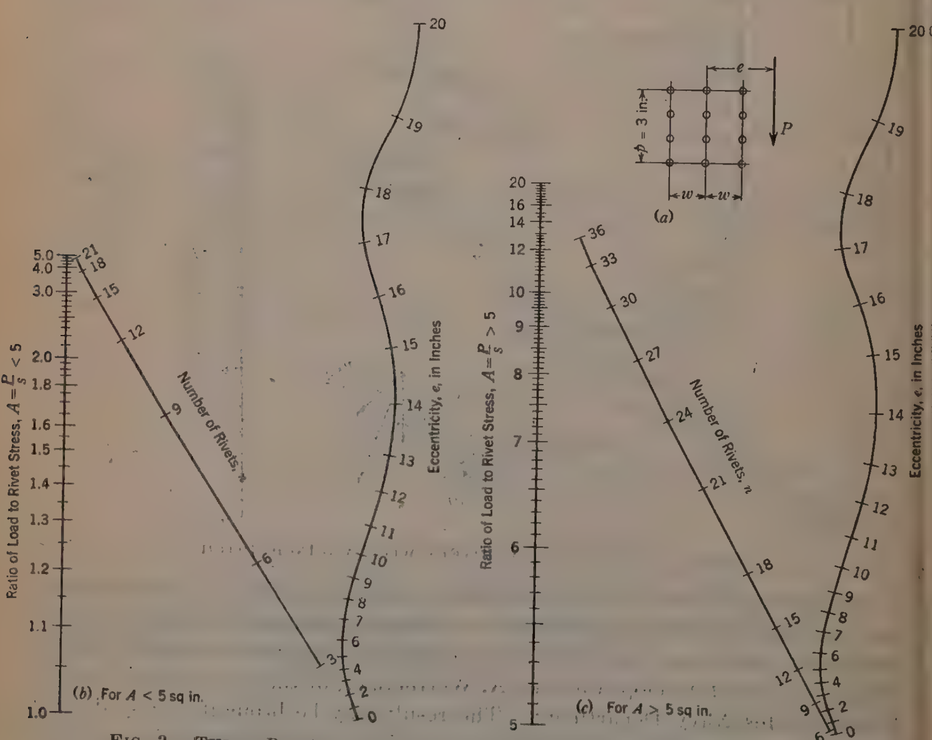

FIG. 3.—THREE ROWS OF RIVETS (p = 3 INCHES AND w = 3 INCHES).

Example 3.—Three Rows of Rivets.—Similar to Equation (11), for the case shown in Fig. 3(a):

$$\frac{1}{A^2} = \frac{1}{n^2} + \frac{e^2 r^2_o}{(\Sigma r^2)^2} + \frac{2 e w}{n \Sigma r^2} \qquad (14)$$

in which,

$$r_o = \left\{ \left[\frac{p}{6} (n - 3) \right]^2 + w^2 \right\}^{\frac{1}{2}} \quad \text{.............} (15)$$

and,

$$\Sigma\, r^2 = \frac{p^2\, n}{108} (n - 3 (n + 3) + \frac{2\, n\, w^2}{3} \quad \text{..............} (16)$$

As before, entering Fig. 3(*b*) with A = 1.23 and e = 10, the broken line intersects the diagonal at slightly more than n = 6, say, n = 9.

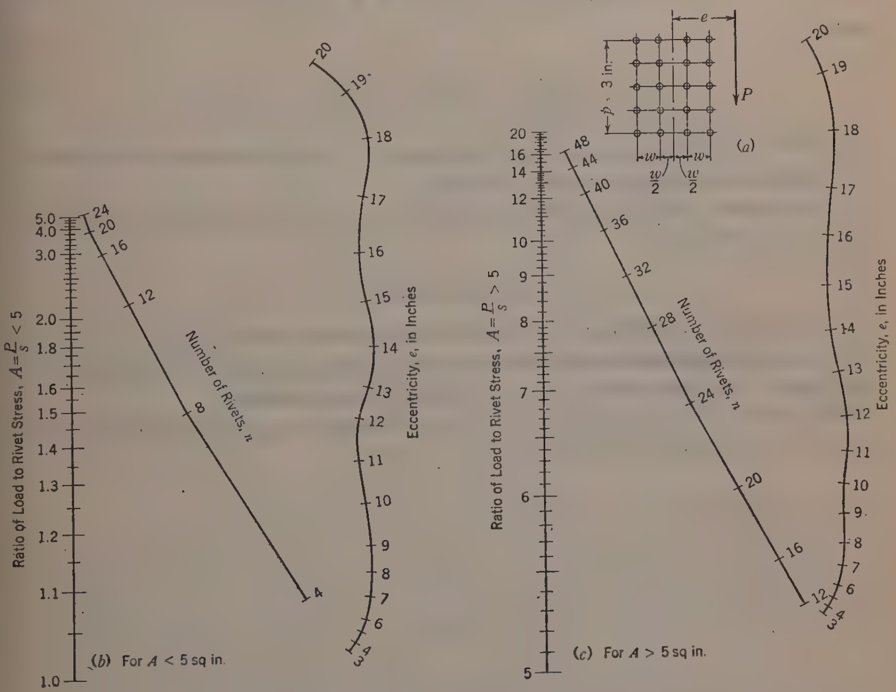

FIG. 4.—FOUR ROWS OF RIVETS (p = 3 INCHES; r_n = 1½ INCHES; AND w = 3 INCHES).

Example 4.—Four Rows of Rivets.—If w_1 = distance between inner rows of rivets, and w_2 = distance between inner row and outer row:

$$\frac{1}{A^2} = \frac{1}{n^2} + \frac{e^2\, r_o^2}{(\Sigma\, r^2)^2} + \frac{2\, e \left(\frac{w_1}{2} + w_2 \right)}{n\, \Sigma\, r^2} \quad \text{.............} (17)$$

in which,

$$r_o = \left\{ \left[\frac{p}{8} (n - 4) \right]^2 + \left(\frac{w_1}{2} + w_2 \right)^2 \right\}^{\frac{1}{2}} \quad \text{.............} (18)$$

and,

$$\Sigma r^2 = \frac{p^2 n}{192} (n - 4) (n + 4) + \frac{n}{2} \left[\frac{w^2_1}{4} + \left(\frac{w_1}{2} + w_2 \right)^2 \right] \quad \ldots . (19)$$

For the case, $w_1 = w_2 = w$:

$$\frac{1}{A^2} = \frac{1}{n^2} + \frac{e^2 r^2_0}{(\Sigma r^2)^2} + \frac{3 e w}{n \Sigma r^2} \quad \ldots\ldots\ldots\ldots . (20)$$

in which,

$$r_0 = \left\{ \left[\frac{p}{8} (n - 4) \right]^2 + \left(\frac{3 w}{2} \right)^2 \right\}^{\frac{1}{2}} \quad \ldots\ldots\ldots . (21)$$

and,

$$\Sigma r^2 = \frac{p^2 n}{192} (n - 4) (n + 4) + \frac{5 n w^2}{4} \quad \ldots\ldots\ldots . (22)$$

Entering Fig. 4(*b*) with $A = 1.23$ and $e = 10$, the broken line intersects the diagonal at about $n = 5$, say, $n = 8$.

The formulas for Examples 3 and 4 are developed in a manner similar to those in Example 2.

Conclusion

The problem of eccentric riveted connections can be simplified by the use of an equation expressing the number and spacing of the rivets required, as a function of the rivet stress, and the load and its eccentricity.

The method and charts presented in this paper can be extended to cover cases other than those illustrated.

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PAPERS

DETERMINATION OF TRAPEZOIDAL PROFILES FOR RETAINING WALLS

By A. J. Sutton Pippard,[1] M. Am. Soc. C. E.

Synopsis

In spite of recent advances in the theory of earth pressures, that due to Rankine is still widely used by engineers in designing profiles for retaining walls. The design is usually a matter of trial and error, a suitable section being guessed and then verified to see that it satisfies the conditions of stress required. A method of direct design is given herein which enables a profile to be determined rapidly. Cases dealt with are those of walls with vertical and sloping backs and with the earth level with the top of the wall, or surcharged. The base width of a suitable wall can be determined quickly for any value of height and top width by the use of diagrams and simple mathematical formulas.

(1) Whatever may be thought of the merits of the Rankine theory of earth pressure in the light of modern research it is still used widely as a basis for design. The object of this paper is to present a convenient and rapid method of determining the profile of a retaining wall, which will just satisfy the usual criteria for strength when it is subjected to an earth pressure calculated by this theory.

The walls considered are trapezoidal in section. The earth surface is either inclined at an angle or level with the top of the wall; that is, the wall may or may not be surcharged.

This problem generally involves either a somewhat laborious computation or a process of trial and error in combination with a graphical analysis. By the help of the diagrams given, the work is reduced considerably.

(2) The late Sir Benjamin Baker, Hon. M. Am. Soc. C. E., has stated[2] that, "Experiment has shown the actual lateral thrust of good filling to be equivalent to that of a fluid weighing about 10 lbs per cubic foot, and allowing for variations in the ground, vibration, and contingencies, a factor of

Note.—Discussion on this paper will be closed in November, 1934, *Proceedings*.

[1] Prof. of Civ. Eng., Univ. of London (Imperial Coll.), London, England.

[2] "The Actual Lateral Pressure of Earthwork," by the late Sir Benjamin Baker, Hon. M. Am. Soc. C. E., *Minutes of Proceedings*, Inst. C. E., Vol. LXV, p. 183.

safety of 2, the wall should be able to sustain at least 20 lbs fluid pressure, which will be the case if $\frac{1}{4}$ of the height in thickness.

"It has been similarly proved by experience that under no ordinary conditions of surcharge or heavy backing is it necessary to make a retaining wall on a solid foundation more than double the above, or $\frac{1}{2}$ of the height in thickness. Within these limits the engineer must vary the strength in accordance with the conditions affecting the particular case."

The assumption of a fluid pressure of 20 lb per cu ft only gives the same result as a design based on the Rankine hypotheses for certain specific combinations of earth density and angle of repose. Thus, if ρ_e is the unit weight of earth fill, ϕ, the angle of repose, and H, the height of the wall, the lateral pressure on the vertical face of a non-surcharged wall is, by the Rankine theory,

$$P = \frac{\rho_e H^2}{2} \left(\frac{1 - \sin \phi}{1 + \sin \phi} \right) \qquad (1)$$

whereas, that on a wall retaining a fluid of density, ρ_L, is,

$$P = \frac{\rho_L H^2}{2} \qquad (2)$$

For these to be equal it is necessary that,

$$\rho_L = \rho_e \left(\frac{1 - \sin \phi}{1 + \sin \phi} \right) \qquad (3)$$

This is a general relationship, but if ρ_L is made equal to 20 lb per cu ft, Equation (3) becomes,

$$\frac{1 - \sin \phi}{1 + \sin \phi} = \frac{20}{\rho_e} \qquad (4)$$

and for any value of ρ_e there will be a corresponding value of ϕ, which will give the same pressure as that calculated from Equation (1).

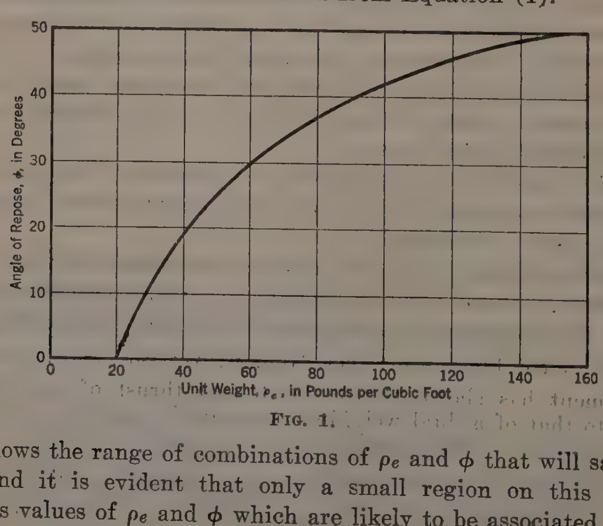

FIG. 1.

Fig. 1 shows the range of combinations of ρ_e and ϕ that will satisfy Equation (4), and it is evident that only a small region on this curve gives simultaneous values of ρ_e and ϕ which are likely to be associated in practical material.

(3) This method of viewing the problem can be used more generally, however, since for any values of ρ_e and ϕ, a value of ρ_L can be found from Equation (3), which will give exactly the same "Rankine" pressure as the earth considered.

This equation can be written in the form,

$$\log \rho_L = \log \rho_e + \log \left(\frac{1 - \sin \phi}{1 + \sin \phi}\right) \qquad (5)$$

which lends itself to simple nomographic representation as shown in Fig. 2.

If a line is drawn on this diagram joining the known values of ρ_e and ϕ on the two outsides lines, its intersection with the middle line gives ρ_L at once. As will be shown later, however, there is no need to calculate the value of P from Equation (2) because a suitable profile can be obtained directly from the value of ρ_L thus found.

(4) The case of the surcharged wall is not quite as simple as that just dealt with but, nevertheless, can be exhibited in a similar form. If α is the angle of surcharge, the earth pressure acting at one-third the height from the base is, according to the Rankine theory,

$$P = \frac{\rho_e H^2}{2} \cos \alpha \frac{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi}}{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi}} \qquad (6)$$

and this pressure is assumed to act parallel to the earth surface. The unit weight of the equivalent liquid is now,

$$\rho_L = \rho_e \cos \alpha \frac{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi}}{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi}} \qquad (7)$$

and, if $\dfrac{\cos \phi}{\cos \alpha} = a$, this becomes,

$$\frac{\rho_L}{\cos \phi} = \frac{\rho_e}{a} \left(\frac{1 - \sqrt{1 - a^2}}{1 + \sqrt{1 - a^2}}\right) \qquad (8)$$

Writing Equation (8) in the logarithmic form:

$$\log \frac{\rho_L}{\cos \phi} = \log \rho_e + \log \frac{1 - \sqrt{1 - a^2}}{a\,(1 + \sqrt{1 - a^2})} \qquad (9)$$

which can also be expressed as a nomogram as shown in Fig. 3.

To use this diagram the value of $\dfrac{\cos \phi}{\cos \alpha}$ is first calculated, and the point on the right-hand line corresponding to it is joined by a straight line to the value of ρ_e on the left-hand line. The intersection of this connecting line with the middle line of the diagram gives $\dfrac{\rho_L}{\cos \phi}$, and this needs only to be multiplied by the value of $\cos \phi$ already found, to give ρ_L.

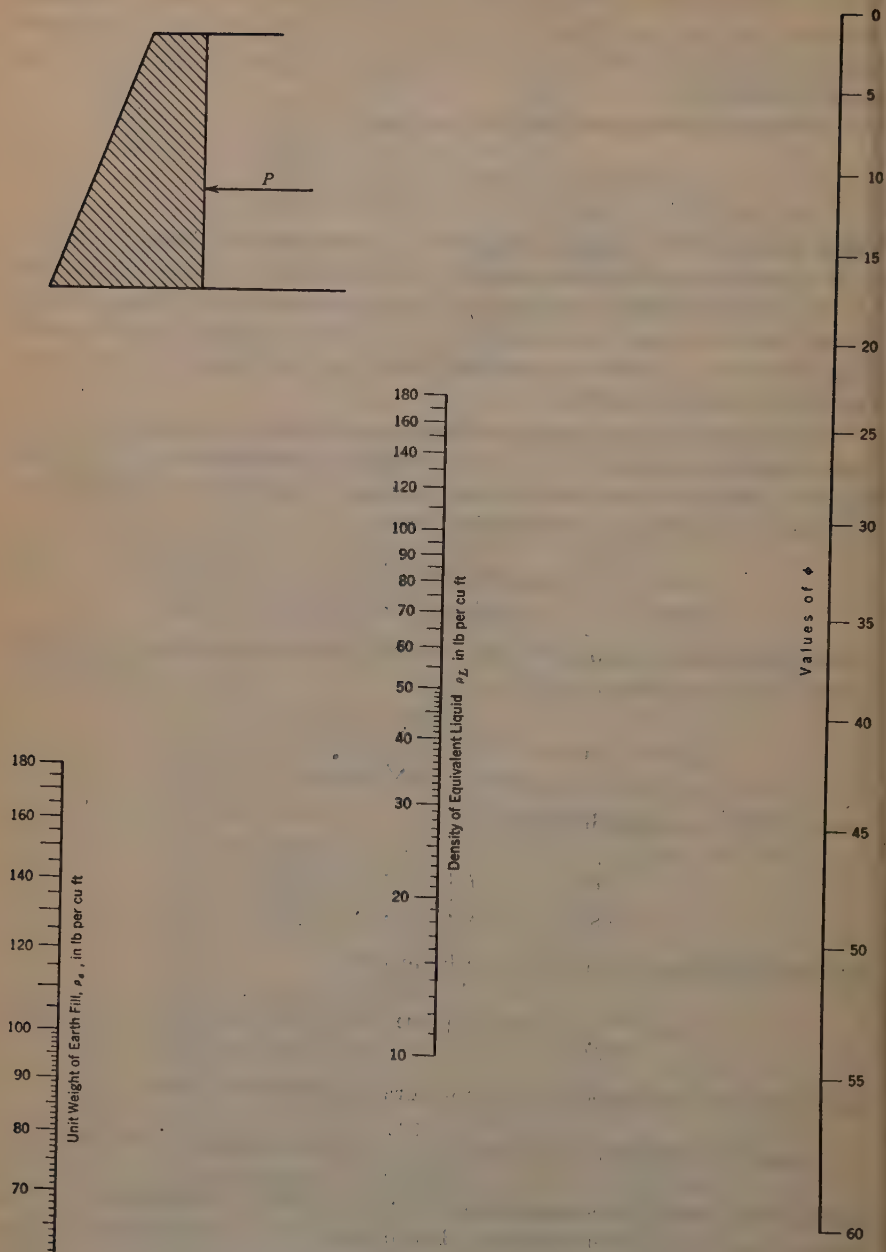
P
Unit Weight of Earth Fill, ρ_e, in lb per cu ft
180
160
140
120
100
90
80
70
60
Density of Equivalent Liquid ρ_L in lb per cu ft
180
160
140
120
100
90
80
70
60
50
40
30
20
10
Values of ϕ
0
5
10
15
20
25
30
35
40
45
50
55
60

FIG. 2.

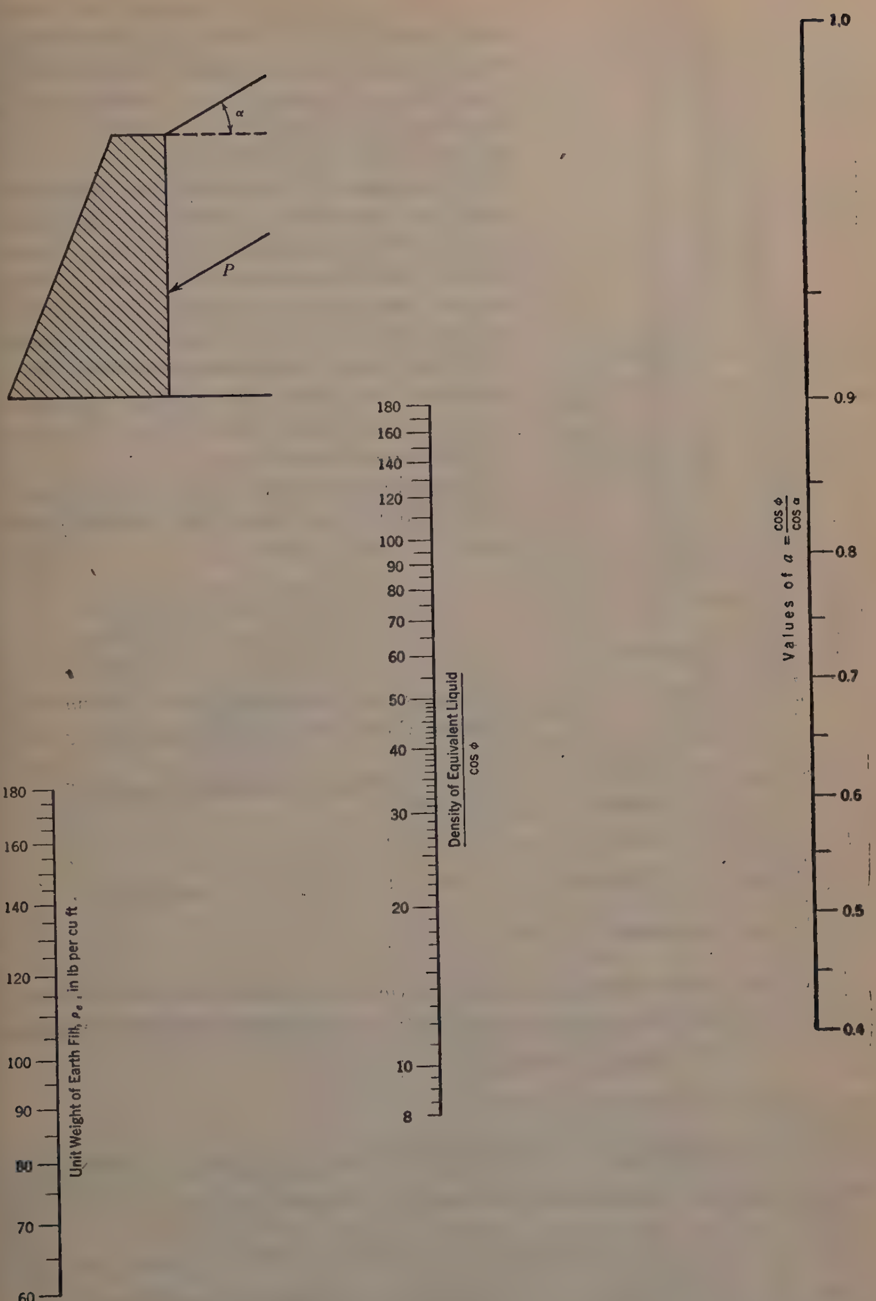
α
P
180
160
140
120
100
90
80
70
60
Unit Weight of Earth Fill, ρe, in lb per cu ft
180
160
140
120
100
90
80
70
60
50
40
30
20
10
8
Density of Equivalent Liquid / cos φ
1.0
0.9
0.8
0.7
0.6
0.5
0.4
Values of a = cos φ / cos α

FIG. 3.

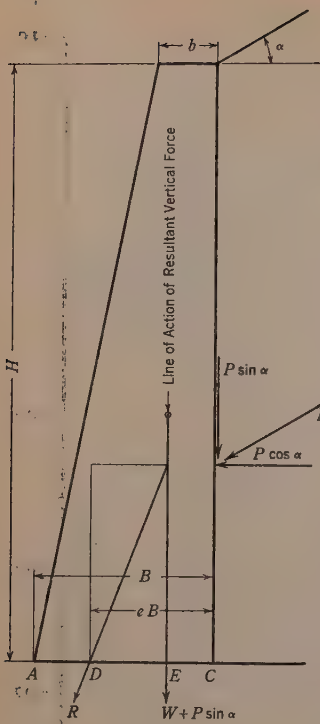

FIG. 4.

(5) The value of ρ_L having thus been found either from Fig. 2 or Fig. 3 (as appropriate), the profile of the wall must be determined. Fig. 4 shows the section of a trapezoidal wall of height, H. The known top width is b and the base width, to be determined, is B.

The pressure, P, acts at one-third the height from the base and is parallel to the earth surface. The resultant of the inclined pressure, P, and the weight of the wall, acting through the center of gravity of the profile cuts the base joint at Point D, a distance, $e\,B$, from Point C, the heel of the wall. It is convenient to resolve the inclined pressure, P, into a horizontal force, $P \cos \alpha$, and a vertical force, $P \sin \alpha$.

By taking moments about C, the distance of the center of gravity of the vertical forces from C is found to be:

$$\overline{C\,E} = \frac{B^2 + B\,b + b^2}{3\,(B + b) + \dfrac{6\,P \sin \alpha}{\rho_m\,H}}$$

in which, ρ_m is the unit weight of the masonry of which the wall is built. Then, $\overline{D\,E} = e\,B - \overline{C\,E}$; and, $\dfrac{\text{total vertical force}}{P \cos \alpha} = \dfrac{H}{3\,D\,E}$. When P is made equal to $\dfrac{\rho_L\,H^2}{2}$, this leads to the result:

$$B^2\,(3\,e - 1) + B\,b\,(3\,e - 1) - b^2 + 3\,e\,\sigma\,B\,H \sin \alpha = \sigma\,H^2 \cos \alpha \quad \ldots(10)$$

in which, σ is the ratio, $\dfrac{\rho_L}{\rho_m}$.

(6) If the usual condition is adopted, that there shall be no tensile stress at the back of the wall, e must be two-thirds, and Equation (10) then becomes:

$$B^2 + B\,(b + 2\,\sigma\,H \sin \alpha) - (\sigma\,H^2 \cos \alpha + b^2) = 0$$

and the solution of this equation is:

$$2\,B = -\,(b + 2\,\sigma\,H \sin \alpha) + \sqrt{5\,b^2 + 4\,\sigma\,H\,(H \cos \alpha + \sigma\,H \sin^2 \alpha + b \sin \alpha)} \; .(11)$$

If b is made zero, the profile of the wall is a triangle and,

$$\frac{B}{H} = \sqrt{\sigma^2 \sin^2 \alpha + \sigma \cos \alpha} - \sigma \sin \alpha \quad \ldots\ldots\ldots\ldots(12)$$

With $\alpha = 0$, Equation (12) reduces to the well-known result for a non-surcharged wall:

$$B = H\sqrt{\sigma} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

The family of curves plotted in Fig. 5 enables $\frac{B}{H}$ to be found for any values of σ and α, and the profile of a triangular wall, to satisfy the Rankine theory, is thus determined directly from the value of ρ_L found from Fig. 3 without the necessity of calculating the value of P.

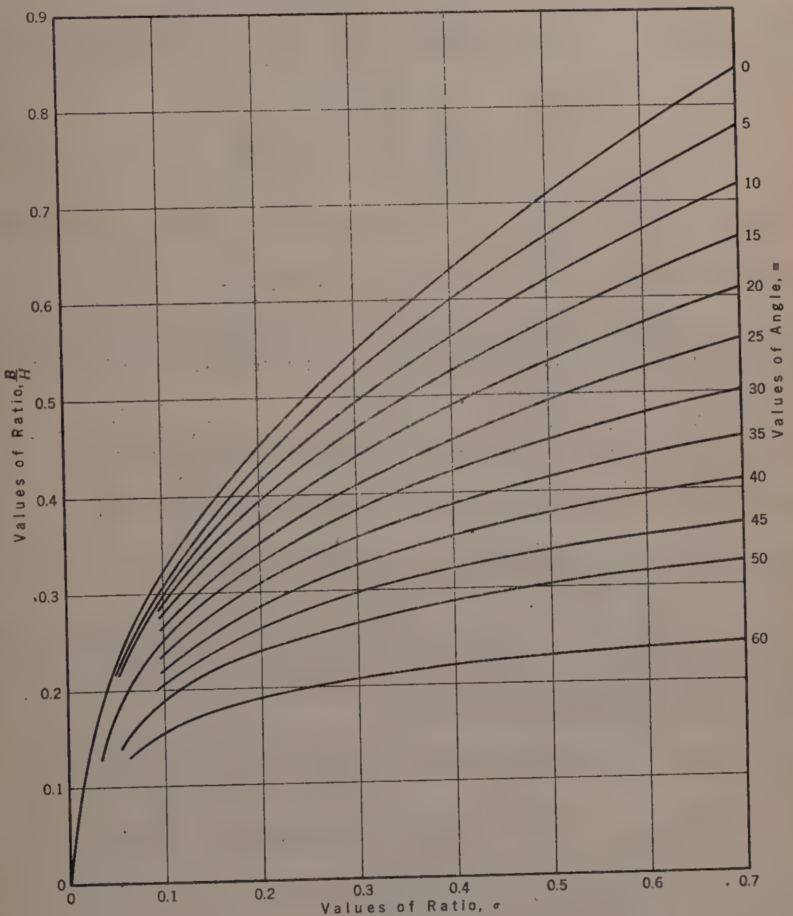

FIG. 5.

(7) To determine the trapezoidal profile of top width, b, proceed as follows: Let B_b be the necessary width of the base of the trapezoidal section given by Equation (11), and B, the base width of the triangular wall, which would satisfy the conditions and which can be found as explained previously. Then, the reduction in the base that can be made, due to the provision of a top width, is, $B - B_b = B'$. Substituting from Equations (11) and (12):

$$\frac{B'}{H} = \frac{n}{2} - \frac{1}{2}\sqrt{4\,\sigma\,(\cos\alpha + \sigma\sin^2\alpha)}\left\{\sqrt{\frac{5\,n^2 + 4\,\sigma\,n\sin\alpha}{4\,\sigma\,(\cos\alpha + \sigma\sin^2\alpha)} + 1} - 1\right\}$$

in which, n is written for $\frac{b}{H}$. Expanding the root in the bracket and retaining only the first two terms as giving a sufficiently accurate result:

$$\frac{B'}{H} = \frac{n}{2} - \frac{1}{8}\left(\frac{5\,n^2 + 4\,\sigma\,n\sin\alpha}{\sqrt{\sigma^2\sin^2\alpha + \sigma\cos\alpha}}\right)$$

Substituting for $\sqrt{\sigma^2\sin^2\alpha + \sigma\cos\alpha}$ from Equation (12) this reduces to,

$$B' = \frac{b}{2}\left(\frac{\frac{B}{H} - \frac{5\,b}{4\,H}}{\frac{B}{H} + \sigma\sin\alpha}\right) \quad \text{.......................(14)}$$

In this connection, $\frac{B}{H}$ is taken from the curves of Fig. 5. Owing to the omission of terms in the expansion of the root of the equation for $\frac{B'}{H}$, this correction is slightly smaller than an exact result would give, but the small error introduced is negligible in view of the nature of the problem and, in any case, is on the right side.

(8) Rankine suggested that the resultant action in the case of earth retaining walls might be allowed to cut the base within the middle three-eighths instead of the middle third. This makes $e = \frac{11}{16}$ in Equation (10), and it is found that the resulting base width of the wall is 3% less than that determined by the curves in this paper.

(9) As an example of the method suppose a profile is required for a wall 20 ft high, retaining earth weighing 120 lb per cu ft and having an angle of repose of 45 degrees. The surcharge angle is 30°, the weight of the masonry, 140 lb per cu ft, and the top width, 2 ft.

Since $\cos\alpha = 0.866$ and $\cos\phi = 0.707$, $\frac{\cos\phi}{\cos\alpha} = 0.816$. Then, from Fig. 3, $\frac{\rho_L}{\cos\phi} = 39.5$, and $\rho_L = 39.5 \times 0.707 = 27.9$ lb per cu ft. Then,

$$\sigma = \frac{\rho_L}{\rho_m} = \frac{27.9}{140} = 0.199$$

Thus, from Fig. 5, the value of $\frac{B}{H}$ for a triangular wall is 0.328; that is, $B = 0.328 \times 20 = 6.56$ ft. Since the top width is 2 ft, a correction may be applied as found by Equation (14), or,

$$B' = \frac{0.328 - 0.125}{0.328 + 0.098} = 0.48$$

and the base width is $6.56 - 0.48 = 6.0$ ft, say. It will be noticed that this thickness is well within the limits suggested by the late Sir Benjamin Baker quoted in Section (2).

This profile is shown in Fig. 6 and a graphical analysis has been made to check it; thus, the weight of the wall = 4.0 × 20 × 140 = 11 200 lb per lin. ft. The earth pressure, by Equation (6), is equal to,

$$\frac{60 \times 400 \times 0.866 \times 0.366}{1.366} = 5\ 560 \text{ lb. per ft}$$

The resultant of these forces is shown in Fig. 6 and cuts the base just inside the middle third, thus checking the result obtained by the use of the curves.

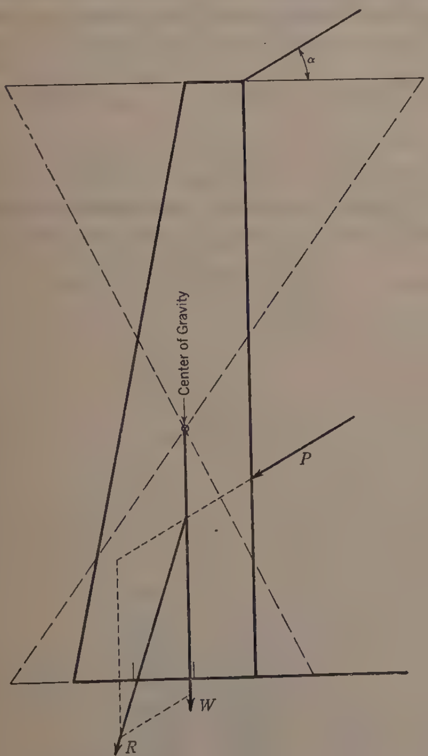

FIG. 6.

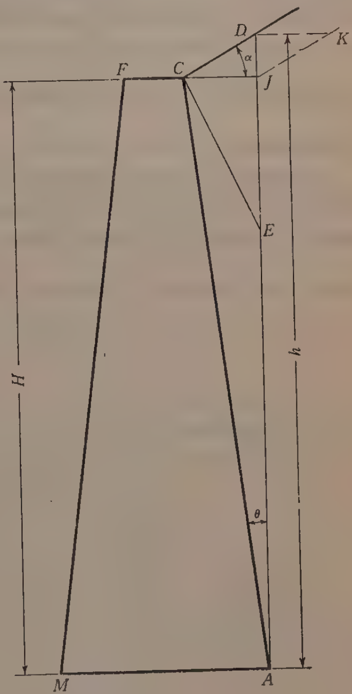

FIG. 7.

(10) When the back of the wall is battered, the problem is more complicated, but the treatment to be given has the advantage of being a direct design method which is easily applied. In Fig. 7, let $A\ C$ be the back of the wall battered at an angle, θ, to the vertical. Then, the resultant action on the back, $A\ C$, is compounded of the weight of earth represented by $C\ D\ A$ and the earth pressure on $A\ D$, of height: $h = H\ (1 + \tan\alpha \tan\theta) = m\ H$.

If $A\ E$ is made equal to $r\ h$, in which, r is the ratio, $\frac{\rho_e}{\rho_m}$, the masonry triangle, $C\ E\ A$, is equivalent to the earth triangle, $C\ D\ A$, as regards both weight and moment about A. Hence, the section, $F\ C\ E\ A\ M$, is the equivalent wall to resist the earth pressure on the face, $D\ A$.

The pressure on $D A$ is proportional to $\rho_e\, m^2\, H^2$, and is equal in magnitude to the pressure that would be exerted on $J A$ if the filling had a density, $\rho_e' = \rho_e\, m^2$, and was surcharged to the angle, α. The earth surface for this equivalent filling is $J K$ in Fig. 7.

The procedure adopted is to design a wall of trapezoidal profile, $F J A M$ (Fig. 7), by the methods previously explained, and to correct the base width thus found to allow for the inaccuracies involved. These inaccuracies are as follows: (a) The absence of the masonry triangle, $C J E$, from the section reduces the weight of the wall from that designed, and also moves the center of gravity away from the back; and, (b) the center of pressure of the force acting on $D A$ is at a height of $\frac{h}{3}$ from A instead of $\frac{H}{3}$, as assumed in the design.

The effects are small and can be dealt with as follows: In Fig. 8, let l be the height of the center of pressure above Point A; k, the distance of the center of gravity of vertical forces from Line $A J$; and, V, the total vertical force acting through the center of gravity $= \rho_m \left(\frac{B + b}{2}\right) H + P \sin \alpha$.

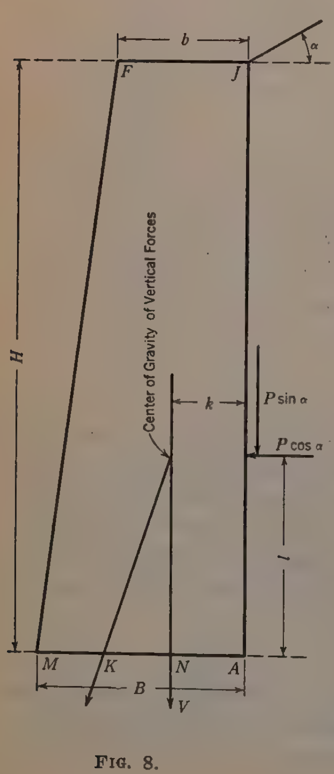

FIG. 8.

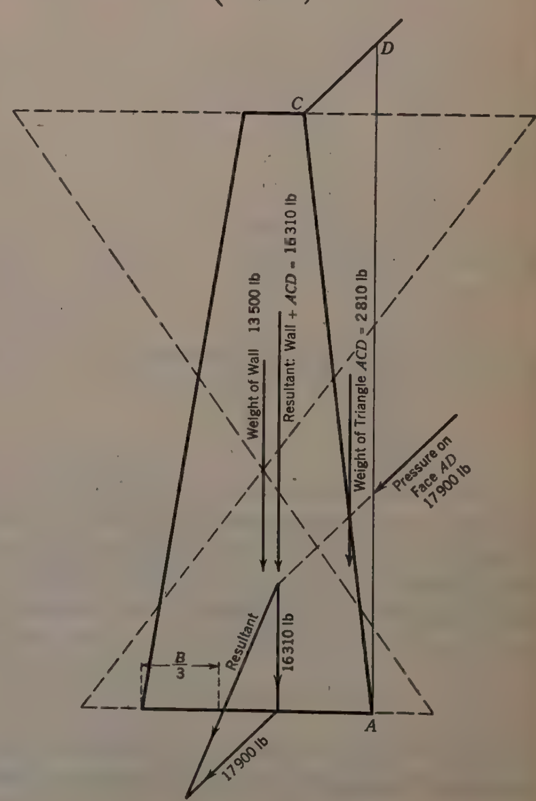

FIG. 9.

Taking moments about Point K:

$$l\,P\cos\alpha = V\left(\frac{2\,B}{3} - k\right)$$

since $\overline{MK} = \frac{B}{3}$. The effect of small changes in V, l, and k, on B is given by:

$$\delta\,B = \frac{\partial\,B}{\partial\,V}\,\delta\,V + \frac{\partial\,B}{\partial\,l}\,\delta\,l + \frac{\partial\,B}{\partial\,k}\,\delta\,k$$

in which, $\frac{\partial\,B}{\partial\,V}$, $\frac{\partial\,B}{\partial\,l}$, and $\frac{\partial\,B}{\partial\,k}$ are the partial differential coefficients of B with respect to V, l, and k, and $\delta\,V$, $\delta\,l$, and $\delta\,k$ are the small increments of these quantities. It follows that $\frac{\partial\,B}{\partial\,V} = \frac{3\,k - 2\,B}{2\,V}$; $\frac{\partial\,B}{\partial\,l} = \frac{3\,P\cos\alpha}{2\,V}$; and, $\frac{\partial\,B}{\partial\,k} = \frac{3}{2}$. In the present case:

$$\delta\,V = -\frac{\rho_m}{2}\,(1 - m\,r)\,H^2\tan\theta$$

$$\delta\,l = \frac{H}{3}\,(m - 1)$$

$$\delta\,k = \frac{\rho_m\,(1 - m\,r)\,H^3\tan^2\theta}{6\,V}$$

$$k = \frac{\rho_m\,H\,(B^2 + B\,b + b^2)}{6\,V}$$

$$l = \frac{H}{3}$$

and,

$$\frac{P\cos\alpha}{V} = \frac{\overline{KN}}{\overline{GN}} = \frac{2\,B - 3\,k}{H}$$

Substituting these values and reducing;

$$\frac{\delta\,B}{H} = \frac{\tan\theta}{2}\left[\frac{(1 - m\,r)\left(\tan\theta + \dfrac{2\,B - 3\,k}{H}\right)}{\sigma'\sin\alpha + \dfrac{B}{H} + \dfrac{b}{H}} + \frac{2\,B - 3\,k}{H}\tan\alpha\right] \quad \text{..(15)}$$

in which, σ' is the value of σ for the equivalent filling; that is, $\frac{\rho_L}{\rho_m}\,(1 + \tan\theta\tan\alpha)^2$. The terms in this expression are readily found from the designed profile, $F\,J\,A\,M$ (Fig. 8), and the increase in base width thus determined.

(11) As an example of the method, consider a wall to the following requirements: Height, H, = 20 ft; top width, b, = 2 ft; batter of back = 1 in 8 (that is, $\theta = 7°.1$); weight of masonry, ρ_m, = 140 lb per cu ft; weight of earth fill, ρ_e, = 100 lb per cu ft; angle of repose, $\phi = 45°$; and, angle of surcharge, α = 45 degrees.

The necessary width of base, B, is to be determined. The equivalent unit weight of earth is $\rho_e' = 100\,(1 + \frac{1}{8})^2 = 126.5$ lb per cu ft. In the first place, the curves are used as in the example of Section (9) to find the necessary width of a wall with a vertical back and a top width of $(2 + H \tan \theta)$ = 4.5 ft. From Fig. 3, $\rho_L = 126.5 \cos \theta = 89.5$ lb per cu ft, and $\sigma' = \frac{89.5}{140}$ = 0.64. From Fig. 5, $\frac{B}{H}$ for a triangular wall = 0.36, or B = 7.2 ft. The correction to be applied for the width is, from Equation (14): $B' = \frac{4.5}{2}\left(\frac{0.36 - 0.281}{0.36 + 0.452}\right) = 0.22$ ft. The base width of the vertical back wall is thus: 7.2 − 0.22 = 7.0 ft, say. The pressure of earth acting at 45° on the vertical back is, $P \doteq \frac{\rho_e' H^2}{2} \cos \alpha$ = 17 900 lb per ft of wall. The vertical component of this is, 17 900 × 0.707 = 12 650 lb per ft. The weight of the wall is 140 × 11.5 × 10 = 16 100 lb per ft. Therefore, V = 28 750 lb and,

$$k = \frac{\rho_m\, H\, (B^2 + B\, b + b^2)}{6\, V} = \frac{140 \times 20 \times 100.75}{6 \times 28\,750} = 1.64 \text{ ft}$$

which, if desired, could have been found graphically.

Let $\tan \theta = \frac{1}{8}$; $(1 - m\,r) = 1 - \frac{5}{7}\,(1 + \tan \alpha \tan \theta) = 1 - \frac{5}{7}\,(1.125)$ = 0.196; $\frac{B}{H} = 0.35$; $\frac{b}{H} = 0.225$; and $k = 1.64$. Substituting these values in Equation (15): $\frac{\delta\, B}{H} = 0.034$; or, $\delta\, B = 0.68$ ft. The necessary base width is, thus, 7.0 + 0.68 = 7.68 ft.

As a check, this profile (shown in Fig. 9) will be examined graphically. The weight of the wall per foot of run is (2 + 7.65) 10 × 140 lb = 13 500 lb. The weight of the wedge of earth resting on the battered back is (2.5 × 22.50) 50 = 2 810 lb. The line of action of the resultant of these two vertical forces is found graphically. The force acting on the vertical earth back is, $P = \frac{\rho_e\, h^2}{2} \cos \alpha = 50 \times 22.5^2 \times 0.707$ = 17 900 lb, and is inclined at 45° to the vertical. Combining these forces the resultant is found to cut the base closely to $\frac{B}{3}$ from the toe of the wall as required.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

EARTHS AND FOUNDATIONS PROGRESS REPORT OF SPECIAL COMMITTEE

Discussion

By Clement C. Williams, M. Am. Soc. C. E.

Clement C. Williams,[79] M. Am. Soc. C. E. (by letter)[79a].—The Committee has submitted much theoretical and experimental information relative to soil mechanics that will aid the judgment concerning foundation problems, and it is generally recognized that no formulation of theory can be substituted entirely for educated human judgment in situations as indefinite as those pertaining to foundations. However, a formulation of theory and observations sometimes offers a frame on which to hang practical experience. For example, modern hydraulics, a very respectable science, has been developed largely from semi-rational semi-empirical formulas in which experimental coefficients and factors were used to allow for variable conditions, refinements being introduced from time to time through the years.

Perhaps if soil and foundation problems should be approached in a similar manner, the accumulated knowledge of soil physics, to which the Committee has contributed so significantly, might be precipitated in forms that would be more readily usable in engineering offices without incurring the danger of a "case-hardening" that would stifle further growth in the science. In order to do this, the theory and observed data should be resolved into forms dependent upon soil characteristics which are either known or can be determined readily with simple apparatus.

Two principal questions arise in foundation design as affected by the soil: (1) What unit load may be placed on a given soil without serious settlement? and (2) what will be the settlement of a soil under a prescribed loading?

Note.—The Progress Report of the Special Committee on Earths and Foundations was presented at the Annual Meeting, New York, N. Y., January 18, 1933, and published in May, 1933, *Proceedings*. Discussion on this report has appeared in *Proceedings*, as follows: August, 1933, by Messrs. L. C. Wilcoxen, H. de B. Parsons, William P. Kimball, and T. A. Middlebrooks; September, 1933, by Messrs. Daniel E. Moran, and A. E. Cummings; October, 1933, by Messrs. Edwin J. Beugler, Jacob Feld, George D. Camp, and Charles Terzaghi; November, 1933, by Messrs. John H. Griffith, Harry E. Sawtell, and O. K. Froehlich; December, 1933, by N. J. Durant, Assoc. M. Am. Soc. C. E.; and January, 1934, by Frederic R. Harris, M. Am. Soc. C. E.

[79] Dean, Coll. of Eng., Univ. of Iowa, Iowa City, Iowa.

[79a] Received by the Secretary April 12, 1934.

With regard to the first question, several factors affect the result, involving both the footing surface and the soil characteristics. The writer has pointed out[80] that the area and shape of the footing constitute one of these factors, the relation being that the supportable load, W, varies as:

$$W = A\,p + L\,s \qquad (74)$$

in which, A is the area of the footing, in square feet; p, the elementary compressive resistance; L, the perimeter, in feet; and s, the shear resistance per linear foot around the perimeter. This expression may be changed to the form:

$$W = A\,p + \frac{k'\,s}{R} \qquad (75)$$

in which, R is the semi-mean radius, $\frac{A}{L}$, and $k' = \frac{p}{s}$. Following the discussion previously cited, k' may be taken as $\zeta\,f$, in which, ζ is the ratio of moduli of rigidity of the soil in shear and compression, $\left(\frac{E_s}{E_c}\right)$, and f is the coefficient of internal friction. The expression then becomes $A\,p\left(1 + \frac{\zeta f}{R}\right)$, or, the average unit load, $\frac{W}{A}$, varies as $p\left(1 + \frac{\zeta f}{R}\right)$. John H. Griffith, M. Am. Soc. C. E., has given some observed values of these properties.[81] Average values may be taken at $\zeta = \frac{1}{3}$ for sand, $\frac{1}{4}$ for clay, and $\frac{1}{8}$ for loam. Values of f are available in engineering handbooks.

An analysis of a considerable number of soil loading tests indicates, that the supporting capacity of any soil varies between the lower and upper limits for that soil type in proportion to the density or weight. All minerals of soils have essentially the same specific gravity; hence, the unit weight of the undisturbed soil measures the density with sufficient accuracy. Spreading this range over an average variation, the relation may be stated as $\frac{W}{A}$ varies as $\frac{w - 90}{30}$, in which, w is the weight per cubic foot of a sample of the undisturbed soil, and 90 is taken as the least unit weight of soil that can practically be used to support a load.

The relation, $\frac{1 + \sin\phi}{1 - \sin\phi}$, as the governing theoretical factor in the passive resistance of soils is attributable to Rankine, ϕ being the angle of internal friction, although he adapted it from preceding theory. A normal mathematical procedure would be to set up a formula involving the product of these

[80] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 306.

[81] "Physical Properties of Soils," by John H. Griffith, *Bulletin 101*, Iowa Eng. Experiment Station.

factors, since the supporting capacity varies with each, and to introduce an observed coefficient in order to relate the result to actual conditions.

The formula for the allowable average unit soil load, therefore, would be:

$$\frac{W}{A} = P = P_1 \left(1 + \frac{\zeta f}{R}\right)\left(\frac{w - 90}{30}\right)\left(\frac{1 + \sin\phi}{1 - \sin\phi}\right) \quad \ldots\ldots (76)$$

in which, P_1 is a coefficient involving p and, perhaps, certain other elements.

A comparative study of a number of soil tests as related to successful loadings indicates that P_1 may be taken as 3.0 for practically zero settlement, when P is in kips per square foot. The value of P_1 may be visualized as the unit supporting capacity of a standard soil load under certain standard conditions of loading.

Equation (76) gives a loading range for gravel under a footing 4 ft square, from about 1.2 tons per sq ft—for a porous material weighing 100 lb. per cu ft, and containing sufficient clay to reduce the value of f to 0.4—to 6.5 tons per sq ft for a compact well-graded gravel weighing 120 lb per cu ft and having a coefficient of friction of $\frac{2}{3}$. It gives a range for clay from 0.5 ton per sq ft for a soil weighing 100 lb per cu ft, with a friction coefficient of 0.1, to 6 tons per sq ft for a dense hard clay weighing 130 lb per cu ft and having a friction coefficient of 0.5.

The second query may be resolved in a similar manner. The modulus of compressive rigidity is found to vary roughly with the inverse of the fourth root of the intensity of the pressure, as may be inferred from test data by Professor Griffith cited previously.[81] Owing to the spread of the load through the soil, the intensity of compression varies roughly with $\frac{1}{\sqrt{y}}$, y being the depth. From these facts, the total settlement may be assumed to vary approximately as $T^{\frac{3}{4}}$, in which, T is the thickness of the compressible soil stratum within the range of the depth affected. The depth affected in a uniform soil[80] will be about $\frac{b}{f}\left(\sqrt{\frac{P}{p}} - 1\right)$, in which, p is the intensity of pressure that the soil will sustain within its elastic resistance; P, the actual intensity of the soil load under the footing; b, the least width of the footing; and f, the coefficient of internal friction. An observation that tends to substantiate this relationship is the case of a symmetrical library building resting on a compressible layer of yellow clay varying in thickness from 4 ft at one end of the building to 14 ft at the other. The observed settlement was $\frac{1}{4}$ in. at one end and about $\frac{5}{8}$ in. at the other.

From the foregoing discussion, the settlement would appear to vary inversely with $\left(1 + \frac{\zeta f}{R}\right)$, inversely with $\frac{w - 90}{30}$, and directly with $T^{\frac{3}{4}}$. It also varies directly with P^x, in which, x is an exponent that depends on the degree of saturation of the soil. In general, x is the exponent shown by the settlement curve of a test loading. Data at hand are not sufficient to de-

termine x statistically. However, if x is taken as $1 + \frac{w_a}{w_p}$ for clay soils, in which, w_a is the actual percentage of water present (in terms of the dry weight), and w_p is the percentage at the plastic limit, the results agree reasonably with the test curves, in which the percentages of water are estimated from general descriptions.

The formula for settlement of a foundation, in inches, then becomes,

$$S = C \times \frac{30}{w - 90} \times \frac{R}{R + \zeta f} \times T^{\frac{2}{3}} \times P^{\,1 + \frac{w_a}{w_p}} \quad \ldots\ldots\ldots\ldots (77)$$

in which, C may be taken as a coefficient of compressibility, or the inches of settlement per kip of load, of a standard soil 1 ft thick under a standard load test with no moisture present. The plastic limit is the percentage of water present, in terms of the dry weight, when the soil can be rolled in cylinders $\frac{1}{8}$ in. in diameter. Inasmuch as the term has no significance for sand, w_p may be replaced for sand in the formula by w_s, the percentage present at saturation.

Again, comparing with tests and observed field data, in which percentages of water and weights of material are estimated in most cases from the general descriptions of conditions, the value of C is found to be 0.02 for clay soils, and 0.01 for sand and for hardpan.

For example, according to Equation (77), the settlement under a footing 4 ft square, carrying 3 kips per sq ft on a clay soil weighing 110 lb per cu ft, having a coefficient of internal friction of 0.2, a plastic limit of 12%, with 8% of water actually present, would be 0.4 in. If the moisture should be increased to 24% (near saturation) and all other factors kept constant, the settlement would increase to about 1.4 in.; or, if the load should be doubled, with other conditions constant, the settlement would be about 1.5 in.

If the soft clay under the tank of "Case B," in the Committee's report, is assumed to be supersaturated with 30% of moisture present (having a plactic limit of 12%, and weighing about 98 lb per cu ft, and having a coefficient of friction of 0.1), Equation (77) would predict about 15 in. of settlement.

If the soil within the range of pressure influence consists of two or more different strata, the pressure on each successive stratum can be estimated by assuming a spread governed by the angle of internal friction, the settlement for each layer similarly calculated, and the summation taken as the total settlement at the surface.

In addition to the aforementioned considerations, attention should be called to the fact that the settlement on clay soils varies about as the square root of the time, continuing frequently for some years, as, for example, in the cases cited by the Committee and by Lazarus White,[82] M. Am. Soc. C. E. The settlement of footings on sand and gravel, in which the air and water present are displaced readily, occurs essentially while the load is being applied. Soils that are predominantly clay behave as clay even though considerable quantities of sand and gravel may be present.

[82] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 341.

Unfortunately, the test data and field observations at hand are not sufficiently detailed to afford a basis for determining the empirical factors in Equations (76) and (77) with finality. However, the values assigned give results consonant with scattered observations and Equation (77) should indicate roughly the settlement that may be expected, comparable in accuracy, perhaps, with retaining-wall theory. Both formulas are proposed primarily with a view to crystallizing, if possible, the voluminous, but rather dispersed, soil observations with which the profession has been favored in recent years. They are predicated on the assumption that adequate tests and explorations will be made at the site to determine the characteristics of the soil and the strata involved.

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DISCUSSIONS

MODIFYING THE PHYSIOGRAPHICAL BALANCE BY CONSERVATION MEASURES

Discussion

By Gerard H. Matthes, M. Am. Soc. C. E.

Gerard H. Matthes,[49] M. Am. Soc. C. E. (by letter)[49a].—The author has focused attention on a much neglected phase of water conservation in the United States. In Europe, the evils resulting from disturbing the physiographic balance which exists in any water-shed between erosion, run-off, and the débris-carrying capacity of streams, have attracted considerable attention. The situation on the Rio Grande, described by the author, shows clearly that the time has come for American engineers to look ahead and profit from European experience. This is the more important in view of the large number of flood-control and water conservation projects now under consideration. As far as the writer is informed, little thought appears to have been given, in any of these projects, regarding the ultimate effect of their operation on the stream channels below them.

Channel deterioration is of two distinct kinds:

(*a*) Excessive bed scour in the reach immediately below a reservoir, caused by lack of sufficient débris load to absorb the energy of the released water normally available for débris transportation; this assumes all the bed load and, at least, part of the suspended load to lodge in the reservoir.

(*b*) Loss of cross-sectional area in the lower reaches due to the inability of the reduced stream flow to remove débris accumulations contributed by uncontrolled tributaries emptying below the reservoir. As illustrated by the Rio Grande case, both forms of deterioration, although of opposite types, may occur in the same stream. In fact, the conditions cited under Class (*a*) operate to aggravate those under Class (*b*). In either case, the hydraulic gradient eventually becomes affected, after which restoration to normal carrying capacities for both water and débris cannot be accomplished except at

Note.—The paper by A. L. Sonderegger, M. Am. Soc. C. E., was published in December, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1934, by Messrs. H. H. Chapman, and E. B. Debler; April, 1934, by Messrs. Frank E. Bonner, and C. S. Jarvis; and May, 1934, by Messrs. W. P. Rowe, and J. C. Stevens.

[49] Prin. Engr., Office of Pres., Mississippi River Comm., Vicksburg, Miss.

[49a] Received by the Secretary April 19, 1934.

considerable cost. It is to be noted that the conditions under Class (*a*) are readily checked by providing suitable sill dams, whereas conditions under Class (*b*) are not easily rectified; and if they are not checked at the start they lead to serious losses of channel capacity.

In most projects interference with the physiographic balance, as described, can be minimized if not completely eliminated by careful *a priori* evaluation of soil erosion and run-off conditions on the one hand, and by the débris-carrying capacities of streams on the other hand. The principle involved is that any stream, viewed as a débris carrier, requires for its channel maintenance the flushing action of bank-full, or nearly bank-full, discharges at appropriate times. Furthermore, intelligent operation of reservoirs can provide such discharge if included as an essential feature in project design and operation. The reason why comparatively little channel deterioration has resulted in the Eastern United States from failure to observe these requirements, is that water utilization there is still in its infancy. Most reservoirs are not large enough to prevent major floods from spilling large volumes into the river below, and, therefore, do not materially curtail the flushing action of flood flows. In the arid Western States where for economic reasons water conservation has been extended farther, reservoirs large enough to absorb most of the flood flow are found on many streams. It is in such cases that the effects of channel deterioration are most noticeable. The author has given a clear portrayal of the problem as it affects these Western conditions.

One factor enters into the problem, which heretofore has received little attention. This is the resistance to scour and transportation by water on the part of débris accumulations which have aged and become compacted. Compaction of this kind is much more pronounced in Eastern than in Western streams, a circumstance which tends to mitigate channel deterioration in the latter. While bank-full flow is practically a necessity for removing débris accumulations in Eastern streams, it is probable that much less than bank-full discharge does effect such removal from streams in the arid West. Accurate data on this latter aspect are greatly needed. Certain it is, that the physiographic balance in Eastern streams is dependent on more frequent high rates of discharge than in the arid West, and that the streams of the latter during high stages carry a vastly greater débris load than Eastern rivers in flood.

An important difference appears to lie in the relative degrees of compaction of the débris. In the East, stream flow is perennial, and any materials left to lie undisturbed under water compact at a rapid rate, especially during the summer and fall when organic matter of all kinds is very active, and bed-load movement practically ceases. Agglutination of the sediment particles results, and this, added to the mechanical bond brought about by the puddling action of water which tends to fill the voids between large particles with smaller ones, enhances cohesion to a marked degree. The finer silts and clays compact more firmly than the coarser sands and gravels. In the arid regions, stream beds are dry or partly dry for many months each year. Oxidation of organic matter, sunlight, and wind then operate to reduce compaction. As flood flow in such streams usually subsides rapidly, débris deposits do not lie

submerged long enough for compaction to reach a high stage of development; and this will account for the notoriously mobile character of the materials composing the dry beds of Western arroyos and rivers. This mobility facilitates their transportation when flow resumes.

Deforestation, overgrazing, and soil erosion have received increasing attention of late years, but the necessity of maintaining unimpaired débris-carrying capacity in streams has not come in for the attention that is due it. Evidently, it has not forced itself sufficiently on the attention of those who have the power to correct it. Impairment of natural channels in its incipient stages, is not easily recognized and, like an insidious disease, its symptoms are not manifest until an advanced stage is reached. This makes it all the more important for the Engineering Profession to be alert in this matter.

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DISCUSSIONS

INVESTIGATION OF WEB BUCKLING IN STEEL BEAMS

Discussion

BY R. L. MOORE, ESQ., AND E. C. HARTMANN, JUN. AM. SOC. C. E.

R. L. MOORE,[5] ESQ., and E. C. HARTMANN,[6] JUN. AM. SOC. C E. (by letter)[6a].—Web failures in beams may occur in any one of the following ways: (1) Yielding of the web material; (2) shear or diagonal compression buckling; and (3) buckling at points of load concentrations. The tests reported by Messrs. Lyse and Godfrey, on steel beams with depth-thickness ratios of 70 or less, involved principally the first type of failure. While their observations are a valuable contribution to present knowledge of web behavior, the investigation can scarcely be considered as one dealing with the interesting phenomena of web buckling and its related problems.

This discussion contains a brief description of a series of tests on aluminum alloy I-beams and girders involving web failures of each of the aforementioned types conducted at the laboratories where the writers are employed. Table 2 is a summary of the results obtained on several different sizes of rolled and extruded I-beams of the duralumin type of alloy known as 17S-T. The following are the nominal mechanical properties of this material: Tensile strength, 58 000 lb per sq in.; yield strength, 35 000 lb per sq in.; elongation in 2 in., 20%; modulus of elasticity, 10 000 000 lb per sq in.; shear strength, 35 000 lb per sq in.; and shear yield strength, 20 000 lb per sq in.

All the beams in Table 2 failed in the web. In Tests Nos. 1, 2, 7, 8, and 9, the web failed by buckling as a column over the reaction. In Test No. 10, failure occurred by buckling as a column under the load point. In Tests Nos. 3, 4, 5, and 6, failure occurred by yielding of the material, complicated in the cases of Tests Nos. 3 and 4 by some column buckling of the web. None of the foregoing failures resulted from elastic instability in the web, permanent set being encountered in every case. This is not surprising, however, since the depth-thickness ratios ranged from about 20 for the 5-in. I-beams to about 25 for the 12-in. I-beams. According to the theory of buckling of flat plates

NOTE.—The paper by Inge Lyse, M. Am. Soc. C. E., and H. J. Godfrey, Esq., was published in February, 1934, *Proceedings*. This discusssion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[5] Research Engr., Aluminum Research Laboratories, Aluminum Co. of America, New Kensington, Pa.

[6] Research Engr., Aluminum Co. of America, New Kensington, Pa.

[6a] Received by the Secretary May 11, 1934

in shear, elastic buckling in Alloy 17S-T becomes critical for depth-thickness ratios of 52, or more.[7]

TABLE 2.—RESULTS OF TESTS ON ROLLED AND EXTRUDED ALUMINUM ALLOY I-BEAMS.

Test No.	Nominal moment of inertia, in inches ^{4}	Depth of beam, in inches	Web thickness, in inches	Span, in inches	Distance from load to nearest reaction, in inches	Load at failure, in pounds	Average computed shear stress at failure, in pounds per square inch	Stiffeners
1	12.3	5	0.210	12	6	55 000	26 200	None
2	12.3	5	0.210	24	12	43 400	20 600	None
3	12.3	5	0.220	48	4	43 000	35 700	None
4	12.3	5	0.220	48	8	38 500	29 100	None
5	12.3	5	0.220	48	4	77 000	64 200	Load and reaction
6	12.3	5	0.220	48	8	47 900	36 200	Load and reaction
7	36.1	7	0.260	48	7	53 800	25 300	None
8	50.0	8	0.275	48	8	59 400	22 500	None
9	229.4	12	0.435	72	12	124 000	19 800	None
10	229.4	12	0.430	72	12	138 200	22 400	Reaction

It is interesting to note that most of the failures occurred by column buckling at points of load concentrations. Failures of this type seems to occur when the average compressive stress exceeds the column strength of the web. In computing this stress it has been found that the load or reaction may be considered uniformly distributed over an area equal to:

$$a = t\ (b + h) \qquad (1)$$

in which, in addition to the notation of the paper, b = effective length of bearing-block, in inches. This area should be considered as symmetrically placed with respect to the load or reaction and should be altered in the case of the latter if the overhang of the beam is shorter than $\frac{1}{2}\ (b + h)$. In beams of normal proportions, without stiffeners, column buckling of the web at points of concentration is more likely to occur than any other type of web failure. Messrs. Lyse and Godfrey prevented this behavior, except in the preliminary tests, by means of web stiffeners, end plates, and suitable bearing-blocks at points of concentrated load and reaction.

Test No. 5 (Table 2) is an interesting example of what may be accomplished by the use of stiffeners in increasing the web strength of beams. A load corresponding to an average computed web shear of 64 200 lb per sq in., almost double the nominal shearing ultimate stress, was carried without producing complete failure in the beam. In Test No. 3, without stiffeners, the average stress at failure was only 35 700 lb per sq in. A similar comparison is found in Test No. 4 and Test No. 6, although the strengthening effect of the stiffeners was not nearly so pronounced. Fig. 22 is a view of Specimens Nos. 5 and 6 showing the shearing distortion possible without actual rupture of the metal.

Values of ultimate strength and corresponding average shearing stresses in the web have been given in Table 2. No values have been given for the yield points of the beams because the load deflection curves break over too gradually to permit the selection of values which are consistently satisfactory.

[7] "Strength of Materials," by S. Timoshenko.

Theoretically, beams will begin to yield when either the web or flange stresses first exceed the elastic range of the material, and since the ordinary formulas of mechanics are quite accurate within the elastic range, the load corresponding to the first yielding of a beam can be calculated quite definitely if the

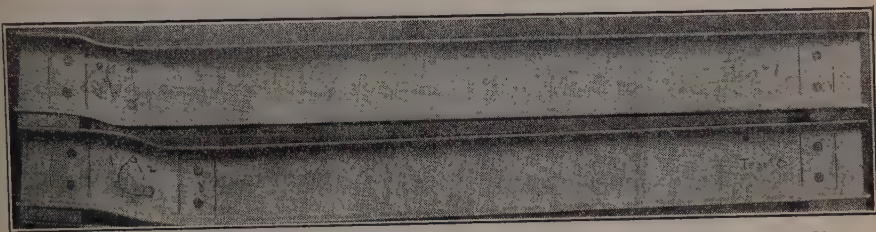

FIG. 22.—WEB FAILURES IN 5-INCH I-BEAMS (TESTS NOS. 5 AND 6, TABLE 2).

stress-strain relations of the material are known in tension, compression, and shear. It should be pointed out in this connection that aluminum alloys have no pronounced yield point such as mild steel and, for this reason, the yielding occurs very gradually and the point of first yield does not represent a distinct change of behavior of a beam under load. The ultimate load-carrying

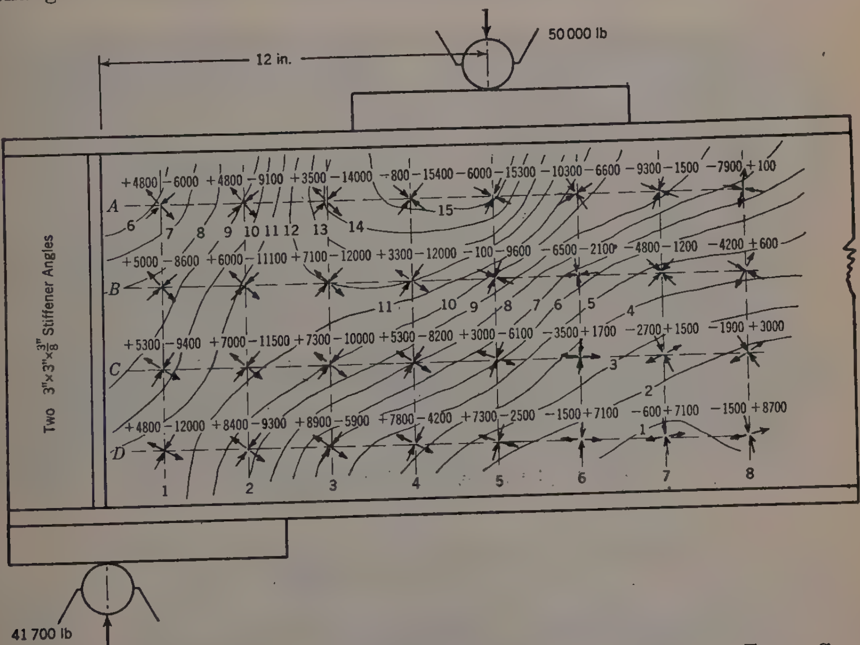

FIG. 23.—MEASURED PRINCIPAL STRESSES IN WEB WITH CONTOURS OF EQUAL COMPRESSIVE STRESS.

capacity of a beam, as determined experimentally, is generally a much more definite value to use in comparison of beams which do not fail by elastic buckling and one which, in the opinion of the writers, is of considerable importance to the designer.

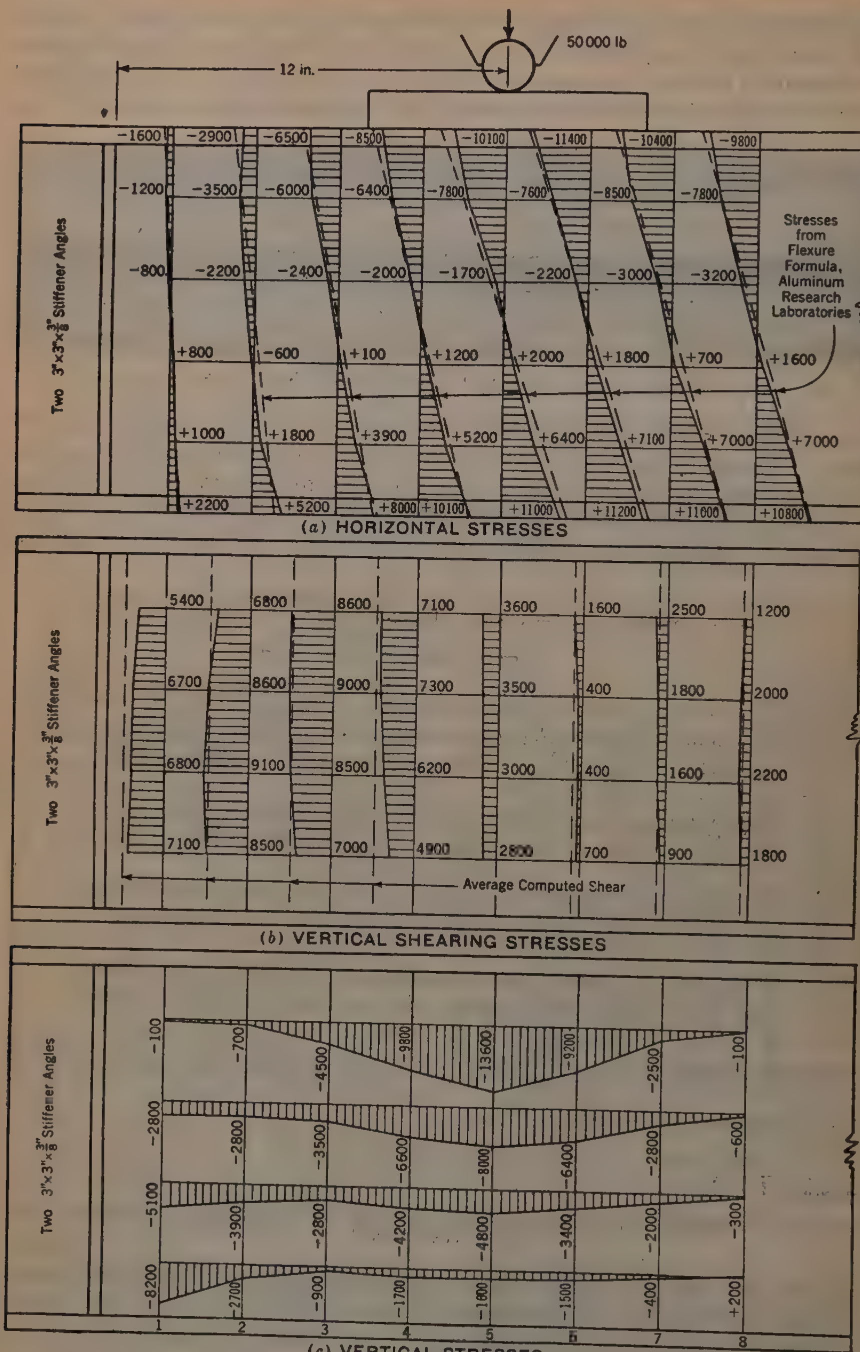

FIG. 24.—MEASURED STRESSES IN THE 12-INCH I-BEAM USED IN TEST NO. 10, TABLE 2 (SEE, ALSO, FIG. 23).

Figs. 23 and 24 show the distribution of stress in the 12-in. I-beam used in Test No. 10. The stress determinations were made by a 2-in. Berry strain-gauge, all measurements on the web being taken on rosettes (four gauge lines intersecting at a point) spaced on $2\frac{1}{2}$-in. centers over the area adjacent to the load and reaction point. All readings were taken on both sides of the web and averaged. In view of the recommendation of Messrs. Lyse and Godfrey that the average web shear be computed on the basis of the net area of the web (h t), attention is called to Fig. 24(b). The agreement there between the measured stresses and the average shear on the gross area (D t) is very satisfactory. The use of net area seems even less justifiable when, as shown for the welded beams of the steel series, it gives average values greater than the maximum as computed by the rational formula.

Fig. 25 shows an aluminum alloy plate girder, 30 in. deep (24 in. clear depth) with a $\frac{1}{4}$-in. web, on which numerous tests have been made in connection with a study of elastic buckling phenomena. Strains and lateral deflections in the web were determined for different positions of load, sizes of bearing-block, and arrangements of vertical and horizontal stiffeners. While the results obtained are too comprehensive to be presented here in detail, it is believed a few observations will be of interest.

FIG. 25.—TEST OF ALUMINUM ALLOY PLATE GIRDER.

When no web stiffeners were used, failures of the web occurred by elastic buckling as a column directly under the load. It was found that the loading required to produce this type of failure could be predicted reasonably well by the same method outlined previously in connection with the discussion of Table 2. This same type of failure also occurred when stiffeners were used and the load was applied between the stiffeners. In such cases it was found, however, that in computing the average compressive stresses the load should be considered uniformly distributed over an area equal to:

$$t\,(b + h)\left[1 + 2\left(\frac{d}{S}\right)^2\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which, s = the clear distance between stiffeners, and the other terms are as previously defined.

When the web was properly stiffened at points of concentrated load, shear or diagonal compression buckling was obtained. Since the lateral deflections occurred gradually, however, almost from the first application of the load, it was difficult to select a point at which the buckling of the web might be said to be critical. In some instances, deflections of as much as $\frac{1}{4}$ in. were observed without permanent set. While this behavior made it impossible to obtain a precise experimental check on the theory of elastic buckling,[7] the writers believe the theory to be satisfactory for predicting loads at which the lateral deflections may become appreciable. It was quite evident, however, that the computed buckling strength of the web was not necessarily a criterion of its ultimate load-carrying capacity.

A double-web box girder of Alloy 17S-T was investigated for web buckling in much the same manner as previously described. The webs in this girder were 15 in. deep by $\frac{1}{16}$ in. thick, having a ratio of unsupported depth to thickness of 172. The results of the tests on this specimen tended to confirm those obtained on the plate girder. It was particularly noticeable in this case that the elastic buckling of the web was not a definite indication of the ultimate load-carrying capacity of the specimen. Even after relatively large lateral deflections were observed, the girder continued to carry higher loads with no evidence of permanent set or other distress. This is a point of considerable interest to designers because there is sometimes a tendency to over-emphasize the importance of elastic buckling. In structures in which the load-carrying capacity considerably exceeds the point of elastic buckling, the writers can see no reason why both factors should not be considered in design.

In concluding this discussion, the following points may be summarized:

1.—In normally proportioned beams, without stiffeners, column buckling of the web at points of load concentrations is more likely to occur than any other type of web failure.

2.—When stiffeners are used at points of concentrated load and reaction, the ultimate load-carrying capacity of a beam may exceed to a considerable extent the point at which yielding of the web material begins.

3.—Since there is no marked yield point in aluminum alloy beams the ultimate load-carrying capacity seems to be the most essential information to be obtained from beam tests in which no elastic buckling occurs.

4.—The measured distribution of stress in the web of a 12-in. I-beam indicates that the average shearing stress should be computed on the basis of gross area of the web rather than on net area, as recommended by Messrs. Lyse and Godfrey.

5.—The buckling of thin webs is a gradual rather than a sudden phenomenon so that it is difficult to obtain a precise experimental check on the theory of elastic buckling. However, the theory apparently provides a satisfactory means of determining the loads at which the lateral deflections may become appreciable.

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DISCUSSIONS

ANALYSIS OF SHEET-PILE BULKHEADS

Discussion

By Messrs. R. L. Vaughn, M. A. Drucker, and Raymond P. Pennoyer

R. L. Vaughn,[16] M. Am. Soc. C. E. (by letter)[16a].—The treatment given in Fig. 7 represents an advanced method of analyzing a bulkhead, which takes into account the restraining action of the ground and the relieving action of the piles back of the bulkhead. It recognizes, as well, the fact that the passive resistance of the soil will be some value greater than that given by the unmodified Coulomb formula. The author himself states that the theory upon which this investigation is based is defective, and devotes much of the remainder of the paper to a demonstration of how hopelessly involved the discussion becomes when an effort is made to develop a better theory.

This is an excellent paper, giving the results of painstaking research and investigation; it presents much information not heretofore generally available in the English language. The writer has found some parts of the mathematical treatment confusing and difficult to follow, and ventures to suggest that the addition of a table of notation would be an improvement.

Perhaps a discussion of a theory of design should not be extended to include a scrutiny of the assumptions as to facts, but most engineers will agree that in any design such assumptions should conform to reasonable expectations. The author presents Fig. 7 as an analysis of the bulkhead at Long Beach, Calif. It becomes in order, therefore, to comment on the asserted physical conditions there shown, as well as to review the admitted defects in theory.

While it is generally referred to as the Coulomb formula, it is understood that the expression, $p = w \tan^2 (45° - \frac{1}{2} \phi)$, is not the exact form originally developed by Coulomb. For many years, however, this formula has been the basis for most discussions concerning earth pressures. The symbol, ϕ, was originally used to designate the angle of repose of a cohesionless granular

Note.—The paper by Paul Baumann, M. Am. Soc. C. E., was published in March, 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: In May, 1934, by Jacob Feld, Assoc. M. Am. Soc. C. E.

[16] Cons. Engr., San Francisco, Calif.

[16a] Received by the Secretary April 16, 1934.

material, and it has been so utilized in many standard texts. Numerous tables of angles of repose of various materials have been published. However, when materials are not cohesionless the term, "angle of repose," becomes meaningless, since the surfaces they will assume, will not be planes and will vary with the height of bank being observed. It has long been recognized that ϕ should be regarded as the angle of internal resistance rather than as the angle of repose. When the internal unit resistance or shear was considered to be the sum of the true friction plus a constant due to cohesion, it was possible to form a conception of ϕ by considering that it was that angle the tangent of which was the shear divided by the normal pressure. At present, it is recognized: (1) That the friction in a given material may not be constant for different depths; (2) that the cohesion varies with the moisture content and probably with the pressure; (3) that the surface of rupture is not a plane, but is dependent on the values of cohesion and friction; and (4) that, in addition to being dependent upon other things, the horizontal pressure against a wall is also some function of the motion of the wall. Recent mathematical discussions have attempted to take all these factors into account and still retain ϕ. Under these circumstances it is difficult to form any conception of ϕ, and certainly it is far from being the angle of repose. The best definition for this angle at present would be that angle which, when used in the Coulomb formula, will give the correct answer. It is suggested that confusion would be avoided if ϕ was reserved for its original meaning and if some other symbol was used in modern formulas.

The method followed by the author to determine the value of ϕ is open to two objections. It has never been proved that the surface angle of repose has any direct relation to the internal pressure, and there is no reason to think that surfaces of repose of samples 6 in. or 1 ft high will be the same as the angle of repose for a bank 20 ft high.

Permanent deformations exhibited by the piling from the test wall indicate that there must have been a considerable degree of restraint at the corners of the tank. This restraint could not be attributed to the copper seals, which appear to have sufficient flexibility, and must have been due to an arching action against the unloaded ground each side of that against which pressure was directly exerted. Any such restraint would cause the wall to appear stiffer than was actually the case and would result in a correspondingly high computed value of the interlock efficiency. This circumstance, together with the fact that the curve assumed by the piling below the surface of the ground must be estimated, would tend to cast some doubt on the experimental determination of the interlock efficiency and also on that of the passive resistance. A further uncertainty is introduced through the arbitrary assumption of the relation between the passive resistance back of the wall and the same resistance in front of the wall.

After having derived a value for the passive resistance in the test case by a process involving assumptions as to the passive resistance back of the wall, assumptions as to the water pressure in the ground, both in front of and back of the wall, and a derived value of the stiffness of the sheet-piling,

the author then derives from the passive resistance thus obtained an "effectiveness factor" to be applied to the Coulomb formula. Aside from the question as to whether or not the passive resistance in the test case has been correctly determined, and aside from the question of using for ϕ values obtained from observations on the surfaces of small samples, the writer knows of no warrant that the same or any other effectiveness factor would be proper for the conditions shown in Fig. 7. If the observations have been made with any approach to accuracy, there must be something wrong about a theory that culminates in a formula the answer to which must be multiplied by a factor of 1.68 or by 2 in order to get the correct result.

Leading authorities now agree that one fault with the Coulomb formula lies in its neglect to include the effect of friction against the face of the bulkhead. This friction only becomes effective apparently when there is an appreciable movement of the wall outward. Equations (16) to (58), inclusive, are presented to show the effect of such friction and, in them, the rather new theory is introduced that ϕ is a function of the displacement of the wall.

That the Coulomb formula, with the addition of an "effectiveness factor" may give questionable results, is evident from the author's actual application of it in Fig. 7. From tests on an experimental box, as described in the paper, it was determined that the effectiveness factor should be 1.68 for this material when it had been deposited in a fill, and for the (assumedly) undisturbed material in the toe slopes it was estimated that a factor of 2 would be suitable. Using that factor, a passive resistance of 20 400 lb per lin ft of wall is obtained for the toe slope at Berth 3. Where the "net passive resistance" of 13 400 lb (referred to in the summary of findings preceding Equation (2)) came from, is not clear, but the former value is the one given in Fig. 7. A rough check on the credibility of this quantity may be obtained by considering resistance against sliding along a plane joining the bottom of the sheet-piles to the nearest point in the bottom of the channel.

The elevation of the bottom of the channel is at − 36.0 (at least, it was agreed on this work to pay for excavation to that depth). The elevation of the bottom of the sheet-piles is at − 38.75; the difference in elevation between the two points is, therefore, 2.75 ft. The horizontal distance between points is 32.5 ft, and the slope is, therefore, $2.75 \div 32.5 = 0.0847$. Using the author's value of 52 lb per cu ft for the weight of the material, submerged, a simple calculation shows that the submerged weight of 1 lin ft of the ideal toe slope at this point is 14 600 lb. Since the prism must be shoved up an adverse grade, the actual weight should be multiplied by the factor, 1.0847, giving an effective weight against sliding of 15 835 lb. Now, if the author's value for the passive resistance of this prism is correct, the coefficient of this material against sliding on itself must be $20\,400 \div 15\,835 = 1.29$. In the construction of hydraulic-fill dams a coefficient of 0.35 is considered safe, although observations have shown that, at times, the coefficient may reach a value of as much as 0.75. Any conclusion to the effect that a fine sand saturated with water can oppose a resistance to sliding equal to one and one-quarter times its own weight seems neither conservative nor reliable.

The author states that American manufacturers advocate the use of 75% of the interlocked section modulus for steel sheet-piles. As far as the writer has been able to learn no such recommendation has been made by American manufacturers except with the qualification that, if such a modulus is to be used, some positive means to prevent slipping along the interlock must be adopted before any load is applied against the piling.

To sum up the objections to the theory upon which the analysis given in Fig. 7 is based: It is assumed that ϕ is a constant and equal to the observed angle of a small sample, whereas there is reason to believe that neither assumption is true. It is assumed that the surface of rupture is a plane, whereas it is generally admitted that such will not be the case. The method takes no cognizance of the effects of friction on the piling. The manner in which the passive resistance is obtained gives a result which, for this class of material, is almost impossibly high. The use of an interlocked section modulus is questionable in the absence of definite and positive means for preventing slippage along the interlock. The author points out many of these objections in his paper.

Turning now to the assumptions of fact. These are two in number: (1) The assumption that the toe slopes would be undisturbed and should retain their theoretical dimensions; and (2) the assumption that the elevation of the water back of the wall would be + 3.5, which is practically mean sea level.

The author states that the toe slopes had been undercut and that in view of the importance of the slopes this was a dangerous procedure. The dredging was conducted in the only practicable manner by which the required channel could be excavated with a cutter type of pipe-line dredge. On the other hand, the design was seriously defective in that the bulkhead had been placed within 32.5 ft of a channel 36.0 ft deep, and that the maximum penetration of the piling was only 2.75 ft below the contract depth in the channel. Especially in view of the unstable and easily erodible character of the material at Long Beach it is not too much to state that any method of excavation would have been dangerous under the circumstances.

The manner in which an hydraulic dredge excavates is not widely known outside of dredging circles and some description is necessary. The entire machine is rotated bodily about a fixed point at the stern, while on successive swings the cutter is lowered to suitable depths, thus removing the material in horizontal layers. There is no method of control by which the depth can be varied with any degree of accuracy as the dredger swings from side to side; that is, the cutter cannot be slid down or up a slope. A floating machine, more than 200 ft long, weighing several thousand tons, controlled by wires, and often operating in rough water, cannot be manipulated with extreme precision; but it is unquestionable that, by a proper adjustment of the width of swing to the depth of cutter, the latter could be placed within a very few feet of the theoretical side-slope line on each swing. At first sight, it would be reasonable to believe that in this manner a practically perfect side slope could be excavated.

However, while the dredge can be swung to any desired width sidewise, it cannot be moved ahead, and as the cutter is lowered on successive swings a nearly vertical bank, circular in plan, is left ahead of the machine. If the material would stand in such a vertical bank until the dredge could be advanced to a position for the next series of swings, there would be no difficulty. Almost invariably, however, the bank will cave down suddenly in a large mass when the cutter has reached some point near the bottom of the cut. Much of this material will fall into a position back of the path of the cutter and cannot be reached. Likewise, some of the material will fall on to the side slopes which have just been formed, will tear loose more or less of the material which has been left there and, with that addition, will also slide down into the cut and come to rest at some point where it cannot be reached. For this reason it is usually found that if an attempt is made to dig the side slopes exactly to line, the result will be a large and unexpected amount of apparent over-dredging there. If the width of swing is narrowed in an attempt to let the material take its own slope, temporarily the slopes will stand at an unnaturally steep angle, only to cave down later and shoal up the channel after the dredge has passed.

One method of overcoming the effect of caving would be to traverse the entire length of channel repeatedly, removing one layer of material at each trip. The ordinary pipe-line dredge is not adapted to such an operation and if it were attempted the output would be greatly reduced and the cost of dredging would be correspondingly increased. No owner would be willing to meet the increased cost and no contractor who contemplated this procedure would ever get any work.

On the other hand, if an attempt is made to finish the channel on the first trip an excessive amount of over-dredging on both sides and bottom is required, as, otherwise, shoaling will occur, which will necessitate a clean-up operation. Consequently, a compromise is reached.

At least one clean-up trip cannot be avoided (except where the cuts are very light) and, therefore, it is just as well to leave enough material to be worth while on the second trip. On the first passage the dredge is swung out over the side slopes and most of the material above the slope line is removed. This operation is necessarily in a series of steps which will be partly above and partly below the slope line. An effort is made to proportion the steps so that the material which remains above the slope line, plus whatever is expected to cave from the bank ahead, will just about fill the holes below the slope line. No effort is made to bring the bottom down to grade; about 4 or 5 ft of material is left for the clean-up.

On the clean-up trip no further dredging is done on the side slopes except at the toe; in fact, the cutter is never raised more than a foot or two above the bottom at any time on this trip. Dredging along the edges of the channel will again disturb the material in the slopes and, at some delayed time, at least some additional material will slide down. If the clean-up is restricted to the specified channel any material that slides down will fall into it, shoaling it along the edges and making another clean-up necessary. It is the

custom, therefore, to swing the cutter a short distance over the line in order to provide a space into which such additional materials may come to rest without encroaching into the required channel. The slopes, when of certain materials, may not reach a condition of final equilibrium for weeks or even months after the clean-up has been made. It is not possible, therefore, to determine with any degree of exactitude, by means of soundings, just how much extra width must be dug; it is necessary to guess at it. If the clean-up is carried too wide there will be much excess excavation for which no payment is received. If the swings are kept too close to the line another clean-up will be necessary. At Long Beach the records showed that, following progress surveys, it was found necessary to return and clean up various portions of the edges of the channel as often as three or four times, in order to obtain the required depth along the lines. Such operations cannot be conducted without some additional disturbance of the material in the slopes.

This stepping of the slopes and slight overcutting at the toe is the undercutting process to which the author refers. There is nothing precise about it, but it is the best that can be done. During an experience extending over a period of twenty-one years the writer has never known a job where this process, or some variation of it, was not used. In that time, he has not seen even one job on which some surplus material had not been removed from the slopes.

In view of the uncertain behavior of all earth, and in view of the unavoidable limitations of the hydraulic dredging process, it is the writer's opinion that it would be most unwise for a designer to rely upon obtaining an undisturbed side slope of specified dimensions along any channel.

Where the purpose of a bulkhead is to retain the fill back of an apron wharf, the temptation is great to place the bulkhead close to the channel line, thereby reducing the width and cost of the wharf. Observation and experience indicate that in most materials it is dangerous to depend upon the embankment above the level of the bottom of the channel for much support unless the horizontal distance between the bulkhead and the channel is at least one and one-half times the vertical distance from top of fill to bottom of channel. Even with such proportions, the field engineer should be prepared to reinforce the embankment with rock at the first sign of weakness.

The second assumption of fact has to do with the water pressure to which the wall was subjected. The method used to determine the water level would have been satisfactory if this had been merely an investigation to determine the stability of the bulkhead at the time the observations were taken, but such was not the case. This analysis was undertaken to determine whether or not the original design had been adequate. The actual water load which the bulkhead would be unavoidably called upon to resist during the construction period is far in excess of that corresponding to the assumed water level. This is the most serious error in the analysis. Even had all assumptions as to section modulus, active pressures, and passive resistances, been fully realized, the bulkhead would have failed if the fill had been placed by the hydraulic method, which is the one specified in the contract. Since, un-

doubtedly, unsuspected water pressures have been at the bottom of many bulkhead troubles, and since the pressures that may arise in an hydraulic fill have been but little discussed and are not widely understood, it follows that this topic is worthy of some consideration.

During the progress of the trial to which the author refers an effort was made to justify the assumed water level on the grounds that a preliminary embankment should have been placed behind the bulkhead and that this, together with the relieving platform, would have prevented any higher level against the wall. In fact, even if suitable material for the purpose had been available, the preliminary embankment would have been of only limited value in this case, while the relieving platform made things much worse instead of better.

There is a tradition in the dredging business that the best thing to do with a weak bulkhead is to pump an embankment behind it before impounding any head of water. The writer has not encountered a completely satisfactory explanation of the manner in which this procedure relieves the pressure against the bulkhead; and it appears that it may not always do so.

The usually accepted explanations are: (*a*) That the materials first deposited have time to settle, consolidate, and develop cohesion; (*b*) that the presence of the embankment will prevent the crowding of any semi-liquid fill against the bulkhead; (*c*) that the natural grading process due to the action of the water leaves the heaviest particles, presumably those with the greatest coefficient of internal resistance, against the bulkhead; and, finally, (*d*) that the embankment "keeps the water away from the bulkhead." Except for the last one these explanations appear rather reasonable.

When discharging at Long Beach, the dredge was pumping at the rate of about 50 cu ft per sec, of which only about 7% was solid material; the remainder was water. When an embankment is pumped against a bulkhead, together with such a surplus of water, it is impossible to believe that the material is not completely saturated, which means, of course, that the water level in the soil will rise to the top of the bulkhead. It follows, therefore (according to the commonly accepted theory), that at the time the embankment is being placed against the structure the latter must be withstanding a full head of water. However, bulkheads have been used, successfully, which obviously could not withstand such a head of water, let alone an additional pressure due to earth. A question naturally arises as to how such a state of affairs could be possible.

The answer to this seeming paradox lies partly in the character of the bulkheads which have been used and partly in a rather obscure phase of the behavior of ground-waters. Imagine, first, a high conical pile of earth resting on a flat impervious surface. Suppose this earth to be saturated and to be kept saturated by a constant addition of water at the top of the cone. According to the theory of water pressures in saturated earth, it might be supposed that the water pressure at the base, immediately under the apex, would be that due to a head of water equal to the height of the cone. Such would not be the case, however, because the water would be free to flow down-

ward and outward through the earth. Any flow of water through the earth results in a relatively great loss of head. The true variation in pressure throughout the cone involves some complicated considerations, but it is obvious that flow would take place and, therefore, full static head would not exist.

Suppose that the cone is divided vertically and that one-half of it is replaced by an impermeable bulkhead. From considerations of symmetry it is evident that the pressure against the bulkhead would be that which had existed along the same plane in the cone. This now represents the condition that exists when pumping is first started to place an embankment back of a wall. As the fill accumulates, the end of the pipe, the apex of the cone, is gradually shifted along the wall until finally a complete embankment has been placed. As the pipe end is shifted there follows beneath it a triangular area of water pressure, the intensity of which varies from a maximum vertically under the pipe to zero at the edge of the cone of saturation. It is evident that the total area of bulkhead subjected to water pressure at one time is limited, so that some support may be gained from adjoining parts of the wall. It is also evident that at no point is there a pressure due to the full height of water above it. In the actual case, conditions are further complicated by the circumstance that the surface supporting the embankment is also permeable, thus permitting a direct downward flow and a further reduction in water pressure. If the supporting ground happens to be saturated, the variation in pressure throughout this ground-water also becomes complicated.

If the bulkhead, instead of being impermeable, has numerous leaks through it there will be an additional and very substantial reduction in pressure corresponding to the increased velocity in the water of saturation occasioned by the escape of water through the leaks. Until recently nearly all bulkheads encountered in the dredging business have been wooden ones which were anything but tight. It has been in connection with such leaky bulkheads that the embankment technique has been developed. This may be a most important point. Some modern types of bulkhead should be nearly impermeable, and it is likely that such structures will not experience the degree of relief from water pressure through the use of an embankment which past experience with leaky bulkheads would indicate. All designers of impermeable bulkheads for use in connection with hydraulic fills should give the subject of water pressure serious consideration.

It is thus seen that, provided no water is impounded, it is possible to pump an embankment back of a wall and yet avoid a full hydrostatic head equal to the height of saturated earth. After the embankment is completed a different set of conditions arise. In the great majority of cases it is necessary to impound water to the full height of the fill before the latter can be completed. At this time, if the bulkhead is water-tight and the embankment placed behind it is permeable, the impounded water will find its own height against the bulkhead, and, as far as water pressure is concerned, the embankment is of no benefit. If the reverse condition obtains, the water can run out through the bulkhead faster than it can flow through the embankment, and there may be little, if any, water pressure against the wall.

The bulkhead at Long Beach belonged to the former class; as designed it should have been practically impermeable, while the sheet-piles had sufficient penetration to impede, greatly, any underflow. The only material available for an embankment was a sand that was relatively permeable. Under the most ideal performance the embankment would have been of limited benefit, and it would have been impossible to avoid the accumulation of substantial heads against some parts of the structure. Certainly, the water would have risen far above mean sea level, the elevation used by the author in his analysis.

In the design, Berths 1, 2, 3, and 12 had been provided with a concrete relieving platform more than 50 ft wide, supported on a veritable forest of piles. The presence of this platform made it impossible to place an effective embankment against the bulkhead. Furthermore, the platform had numerous small openings in it, sufficient to have transmitted, freely, pressure from any ground-water that might have accumulated above it; and it is inconceivable that a water-tight fill could have been deposited against the under side of the platform by any means whatever; there must have been numerous spaces at this point of a capacity sufficient to permit the unobstructed flow of water from points back of the platform to the face of the bulkhead and to have permitted the transmission of pressures through the water filling such spaces.

Relieving platforms are supposed to do what their name implies, relieve pressure against the bulkhead. It seems proper to point out that under some circumstances they may defeat their own purpose. In the present instance, for illustration, suppose that, first, the space under the platform had been filled as well as possible by means of pumping or sluicing, and that then an impermeable fill had been deposited on top of the platform by any suitable means. This produces as good an embankment behind the bulkhead as could be obtained by any reasonable and practicable method. The final step would then be to fill the remainder of the area by pumping. To do this it is necessary to raise the ground-water to the top of the fill. There would be nothing to prevent water finding its way along the under side of the platform to the face of the bulkhead and transmitting full hydrostatic pressure to that point. In the imagined illustration, the platform would be cutting off the pressure due to a fill of about 12 ft, undoubtedly, but it would be letting in the pressure due to 12 ft of water which is much worse. It is suggested, therefore, that in the case of hydraulic fills, relieving platforms may have a most dubious value.

At present, there is a prospect that some rather high débris dams may be built in California. If this should come to pass it is to be hoped that devices for measuring pressures (both of water and of materials) at various depths and at various time intervals will be incorporated in them. If the variety of débris impounded should cover a wide range in different places, then after sufficient observations had been taken it might be possible to evolve an empirical method of determining earth and water pressures which would be, at the same time, reasonably simple and fairly reliable.

M. A. Drucker,[17] Esq. (by letter).[17a]—The experiment on full-sized steel sheet-piling, under field conditions approximating those to which actual structures are subject, should prove of great value when designing this rather recent type of construction. It is of special importance at the present time, since the assumptions upon which designs are based, have recently undergone a radical change. At about the time when, as the author states, high bulkhead walls began to be constructed, the depths of sheeting embedment and the stresses in anchor and sheeting were determined by considering an active pressure greater than, and a passive resistance less than, the theoretical values obtained for them. Professor Wolmar Fellenius[18] recommended that the active pressures be taken as 25% greater, and the passive resistance as 25% less, than their theoretical values. This is quite different from the assumption that the actual passive resistance is twice the theoretical.

The author is quite right in pointing out that the present theory of bulkhead design is defective in that it does not take into consideration the compression of the soil due to the movement of the wall. By taking the effect of this compression into account he obtains a passive resistance, as illustrated by Fig. 14, which is quite at variance with that obtained by following the Brennécke-Lohmeyer method, as illustrated by Fig. 5. Mr. Baumann's method satisfies the main conditions necessary to be met when using the latter theory. These conditions are: (*a*) The moments about any point on the sheeting must balance, and their sum be zero about the bottom; and (*b*) the sum of the loads on one side of the sheeting must equal the sum of the loads on the other side. With these conditions as a basis, the design is then determined by means of a graphical method.

At first glance this method appears to be highly scientific. On analysis, however, it becomes apparent that, in effect, it is based primarily on an arbitrarily assumed point of zero moment below the ground surface which, in turn, determines the passive resistance distribution for the entire depth of embedment. The fact that the moment diagram, based on this passive resistance distribution, shows that the moment is zero at about the same distance below the ground surface, as originally assumed, can be taken as a check on the arithmetical or graphical work, but not as a proof of the validity of the assumption.

In accordance with this method the depth of embedment, t, is obtained from Equation (2), in which, a is the distance from the ground surface to an assumed point of zero moment, and t_0 is equal to $\sqrt{\dfrac{6\,B_0}{2\,p_p - p_a}}$, as will be shown subsequently. The symbols under the radical denote the following: p_p is the theoretical passive soil resistance; p_a, the unit active pressure increment below the ground surface; and B_0, the reaction at the point of assumed

[17] Structural Designer with Board of Transportation, New York, N. Y.

[17a] Received by the Secretary, June 5, 1934.

[18] "Calculation of Docks and Bulkheads Walls," *Engineering Record*, September 20, 1913.

zero moment, then,

$$t = a + 1.2 \times \sqrt{\frac{6\,B_o}{2\,p_p - p_a}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (68)$$

Pennoyer's formula[19] for the necessary depth of embedment is,

$$t = 1.1 \left(a + \sqrt{\frac{6\,B_o}{2\,p_p - p_a}} \right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (69)$$

The value of t, obtained by using Equation (68), is somewhat larger than that obtained from Equation (69). It may be noted that the latter gives the same depth of embedment as would be obtained by simply balancing moments, of all pressures, about the anchor rod, using a passive resistance equal to the theoretical value instead of twice that value.

The following analysis will show that the assumption of zero bending moment at the distance, a, below the original ground surface, or bottom of channel, also determines the passive resistance distribution below Point C, Fig. 16. Furthermore, the only significance of this point is that the active pressure per unit area is equal to the unit passive resistance.

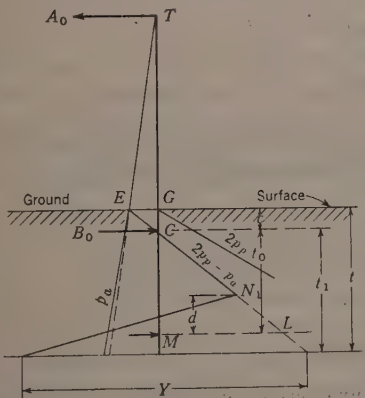

FIG. 16.

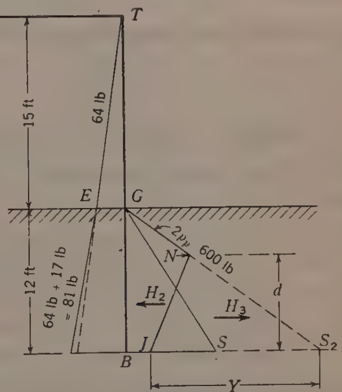

FIG. 17.

In Fig. 16, t designates the total depth of embedment; $t_1 = t - a$; and t_o is that distance below Point C that would make the moment about Point M, of the pressure triangle, MCL, equal to the moment of B_o about that point, or $B_o t_o = (2\,p_p - p_a) \times \frac{t_o^3}{6}$, from which,

$$t_o = \sqrt{\frac{6\,B_o}{2\,p_p - p_a}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (70)$$

It also follows that,

$$B_o = \frac{t_o^2}{6}\,(2\,p_p - p_a) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (71)$$

As the moment at Point C is assumed to be zero, the part of the sheeting designated by t_1 may be treated as a cantilever sheeting of this length, subject

[19] "Design of Steel Sheet-Piling Bulkheads," by Raymond P. Pennoyer, Assoc. M. Am. Soc. C. E., *Civil Engineering*, November, 1933, p. 615.

to a force, B_0, at the top, and embedded in a soil the passive resistance of which is $(2\,p_p - p_a)$. For such a case the passive resistance distribution may be readily determined from the formula,

$$Y = \frac{[(2\,p_p - p_a)\,t_1^2 - 2\,B_0]^2}{(2\,p_p - p_a)\,t_1^3 - 6\,B_0\,t_1} \quad \ldots\ldots\ldots\ldots\ldots (72)$$

which is based on having the horizontal forces, and their moments about any point, balance.

Substituting in Equation (72) the value of B_0 as given by Equation (71) and, also, substituting $\frac{t_1}{1.2}$ for t_0, the value of Y is found to be $1.93\,t_1 \times (2\,p_p - p_a)$, and,

$$\overline{JB} = 0.93\,t_1\,(2\,p_p - p_a) \quad \ldots\ldots\ldots\ldots\ldots (73)$$

If d represents the distance of the point, N_1, above B, then, also, as obtained for cantilever sheeting,

$$d = \frac{(2\,p_p - p_a)\,t_1^2 - 2\,B_0}{Y} \quad \ldots\ldots\ldots\ldots\ldots (74)$$

By making the substitutions indicated,

$$d = 0.40\,t_1 \ldots\ldots\ldots\ldots\ldots (75)$$

If the depth, t, is obtained from Equation (69), then, by making substitutions similar to the foregoing, $\overline{JB}$ and d were found to have the following values:

$$\overline{JB} = 1.60\,t_1\,(2\,p_p - p_a) \ldots\ldots\ldots\ldots\ldots (76)$$

and,

$$d = 0.28\,t_1 \quad \ldots\ldots\ldots\ldots\ldots (77)$$

Although Equations (68) and (69) are based on the same, or on new, principles of bulkhead design, a comparison of the values of $\overline{JB}$, as obtained from Equations (73) and (76)—as well as the values of d from Equations (75) and (77)—show that the pressure distributions are quite different despite the fact that each of the values will satisfy the conditions for such design.

It will now be shown that for any depth of embedment, t, which is greater than that required for the assumed actual unit passive soil resistance, any number of passive resistance distributions may be obtained by varying the position of Point N (Fig. 17), and yet satisfy Conditions (a) and (b). On this basis, however, the moment may not be zero at a distance, a, below the ground surface. The writer has seen no good reason given, however, and none is apparent, why the moment should be zero at that distance below the ground surface. For this analysis, the pressures, length of sheeting, and depth of embedment will be taken the same as given for the author's experiment (see Fig. 17).

The total pressure on the sheeting due to water is $64 \times \frac{27^2}{2} = 23\,300$ lb. The active pressure due to the soil, which has an effective weight of 52 lb, will be taken at 17 lb per sq ft per ft of depth, giving a total pressure of $17 \times \frac{(12)^2}{2} = 1\,220$ lb. The sum of these pressures is 24 520 lb. The sum of their moments about the point, T, is $23\,300 \times \frac{2}{3} \times 27 + 1\,220\,(15 + \frac{2}{3} \times 12)$, or 448 000 ft-lb. If it were assumed that these pressures are resisted at the bottom of the sheeting by a uniformly varying pressure of a total magnitude, H_2, represented by BGS (Fig. 17) then $H_2 = \frac{448\,000}{15 + 0.67 \times 12} = 19\,500$ lb, giving a required unit passive resistance equal to $\frac{19\,500 \times 2}{(12)^2} = 270$ lb. The moment of H_2 about Point $B = 19\,500 \times 4 = 78\,000$ ft-lb. For this condition, then, taking moments about B, $27 A_0 = 23\,300 \times 9 + 1\,220 \times 4 - 78\,000 = 215\,000 - 78\,000$, which equals 137 000 ft-lb, and $A_0 = 5\,070$ lb. This stress in the anchor rod, per foot of wall, is considerably greater than the several values obtained by the author. It is probably much larger than would exist for the case of a steel sheeting. It was obtained here for the purpose of indicating what the anchor stress would be in case the actual passive resistance were not larger than the theoretical value. This large stress may also be taken as an indication of what the anchor stress might be in the case of a rigid bulkhead which condition would be approached if a concrete wall were used. While it seems reasonable to consider that steel sheeting has some flexibility, it does not seem likely that it could be sufficiently strong to withstand the pressure it is subject to and yet be flexible enough to exert a large back-kick at the bottom of the sheeting at the same time that it also exerts a large pressure at the front of the sheeting only a short distance above the bottom, as shown in Fig. 5.

However, assuming that the actual passive resistance is equal to twice the theoretical value, and that the sheeting has some flexibility, Conditions (*a*) and (*b*), previously stated, may be satisfied for any number of passive resistance distributions at the front of the wall for the same depth of embedment. These distributions may be obtained by assuming Point N (Fig. 17) at different distances above Point B.

After assuming the position of Point N, the next step is to determine the value of the negative pressure triangle, JNS_2, represented by H_3, which is that part of the pressure, BGS_2, which does not need to be called into action. The magnitude of this pressure may be obtained by taking moments about Point T; and, after obtaining H_3, the value of A_0 may be obtained by taking moments about Point B, so that, for a passive resistance equal to $2\,p_p$, or 600 lb, the pressure triangle, BGS_2, which will be referred to as H_4, equals $600 \times \frac{(12)^2}{2} = 43\,200$ lb, and its moment about Point T is $43\,200\,(15 + 0.67 \times 12) = 995\,000$ ft-lb. This moment, minus that of active pressures

about the same point, would be equal to the moment of H_3 about that point; or, $H_3\left(27-\frac{d}{3}\right) = 995\,000 - 448\,000 = 547\,000$ ft-lb. For balancing moments about Point T, then, the magnitude of H_3 would depend on d, giving,

$$H_3 = \frac{547\,000}{27 - \frac{d}{3}} \quad\quad (78)$$

Similarly, the stress in the anchor rod, A_0, would depend on the value of d and H_3, and could be obtained by taking moments about B. The moment of H_4 about $B = 600 \times \frac{(12)^3}{6} = 172\,800$ ft-lb, while the sum of the moments of the active pressures, about this point, is 215 000 ft-lb, as previously obtained. Therefore, $27\,A_0 = 215\,000 - 172\,800 + H_3 \times \frac{d}{3} = 42\,200 + H_3 \times \frac{d}{3}$, or,

$$A_0 = \frac{42\,200 + H_3 \times \frac{d}{3}}{27} \quad\quad (79)$$

Furthermore, if $\overline{JS_2} = Y$, then,

$$Y = \frac{2\,H_3}{d} \quad\quad (80)$$

and,

$$\overline{BJ} = 7\,200 - Y \quad\quad (81)$$

To balance horizontal pressures, $H_4 + A_0 - H_3 = 24\,520$ lb, or,

$$H_3 - A_0 = H_4 - 24\,520 = 43\,200 - 24\,520 = 18\,680 \text{ lb} \quad\quad (82)$$

Table 4 gives the values, obtained by means of Equation (78) to Equation (82), for several distances of d arbitrarily assumed. The closeness of the values in Column (7) to 18 680, given by Equation (82), shows that the horizontal pressures balance for all values of d. The moments balance, of course, because

TABLE 4.—Computation of Equation (78) to Equation (82)

Distance, d, in feet	Equation (78)			Equation (79)		Equation (82)	Equation (80)	Equation (81)
	$\frac{1}{3}d$	$27-\frac{1}{3}d$	H_3	$\frac{1}{3}H_3 d$	A_0			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
3........	1	26	21 000	21 000	2 340	18 660	14 000	—6 800
6........	2	25	21 850	43 700	3 180	18 670	7 280	— 80
9........	3	24	22 800	68 400	4 100	18 700	5 070	+2 130
12........	4	23	23 750	95 000	5 070	18 680	3 960	+3 240

the values of H_3 and A_0 in Table 4 (Columns (4) and (6)), were obtained on that basis; so that, while the Brennecke-Lohmeyer method,[20] which is essentially the same as the Blum method,[21] gives a passive resistance distribution that checks mathematically, other possible distributions may be obtained which check likewise. In fact, by assuming a large passive resistance for the soil, and a small value for d, it would be possible to satisfy Conditions (a) and (b) and to obtain not only a small tensile anchor stress, but even a result indicating that the rod is in compression. This would be an absurdity, of course, because even if the sheeting could act as a cantilever, it would deflect away from the soil at the top of the wall.

That it is irrational to assume zero moment at Point C, Fig. 18(a), may be shown in another way. For a passive resistance of $2\,p_p = 600$ lb and an active pressure below the ground surface = 81 lb, $a = \dfrac{960\ (= EG)}{600 - 81} = \dfrac{960}{519} = 1.85$ ft. Therefore, $t_1 = 12.0 - 1.85 = 10.15$ ft.

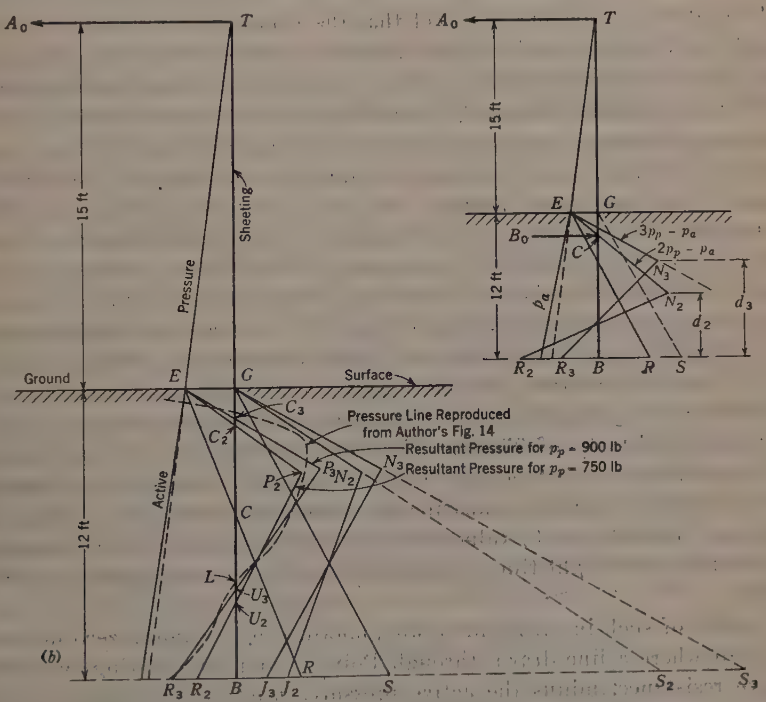

FIG. 18.

Treating the part, TC, as a simple span and taking moments about Point T, B_0 was found to be 5 100 lb, and, after substituting values in Equation (72), BR_2 was found to be 2 780 lb, and from Equation (74), d_2 was found to be 5.4 ft; also, A_0 was found to be 3 020 lb. for this condition.

[20] *Die Bautechnik*, No. 5, 1930.

[21] Carnegie Steel Sheet-Piling, Insert B.

Similarly, for an assumed actual passive resistance of $3\,p_p$, or 900 lb, the following values were obtained: $a = 1.17$ ft; $B_0 = 4\,990$ lb; $BR_3 = 1\,470$ lb; $d_3 = 8.32$ ft; and $A_0 = 2\,780$ lb.

For the purpose of comparison, some of the foregoing values, as well as the corresponding ones previously obtained for a required passive resistance of 270 lb represented by Triangle *BGS* are given in Table 5.

TABLE 5.—Comparison of Stresses

Assumed passive resistance, in pounds	Pressure at Bottom of Sheeting		Anchor stress, A_0, in pounds
	Distance (see Fig. 18(*a*))	Pressure, in pounds	
(1)	(2)	(3)	(4)
270	*BR*	+1 310	5 070
600	BR_2	—2 780	3 020
900	BR_3	—1 470	2 780

It would be reasonable to expect that the values in any column of Table 5 should vary approximately as the assumed passive resistance, or, at least, they should vary consistently. As the values for the passive resistance of 270 lb were obtained by simply balancing moments about Points *T* and *B*, which may be termed the "ordinary method," in which no arbitrary assumptions are involved, these values may be taken as the basis for comparing the others. It is seen from Table 5 that for a passive resistance of 600 lb, the pressure at the bottom of the sheeting is 4 090 lb less than for a passive resistance of 270 lb, while for a resistance of 900 lb, this pressure is 1 310 lb greater than for the resistance of 600 lb. Thus, these pressures do not even vary in the same direction although those for the 600 and 900-lb resistances were obtained by the same method. From Column (3), Table 5, and Fig. 18(*a*), it may be inferred that the higher the assumed passive resistance the nearer will the pressure at the bottom of the sheeting approach that for the passive resistance of 270 lb. This shows clearly that the assumption of zero moment at the distance, *a*, below the ground surface is not reasonable. This may also be seen by comparing the values in Column (4), Table 5. The difference between the A_0 values for a passive resistance of 600 lb and that for 270 lb is about eight times as great as that between the values for 900 and 600 lb.

In view of such inconsistent results, obtained by assuming zero moment at the point where a line drawn through Point *E* and representing the assumed passive resistance, minus the active pressure, reaches the sheeting line, *TB*, and based on analyses which will not be gone into here, the writer suggests that the formula,

$$d = t\,(0.65 + 0.02\,r) \qquad \text{(83)}$$

will give approximately accurate passive resistance distribution for any assumed unit actual passive resistance.

In Equation (83), d is the distance from the bottom of the sheeting to the point, N, and r is the ratio of the assumed unit passive resistance to that unit resistance for which a uniformly varying pressure from Point G to Point B would balance the moments about Point T. Generally, the latter resistance is the theoretical passive resistance of the soil.

In Fig. 14, Mr. Baumann gives the pressures, moments, and deflections based on his analysis of passive resistance as indicated in Fig. 11. In the paragraph preceding "Conclusions," the author states that he found it necessary to apply a factor of 2.5 for the passive resistance of the soil, in order to establish equilibrium. This factor would give a passive resistance of 2.5×300, or, 750 lb. As the unit passive resistance required for a uniformly varying resistance for the entire depth of embedment was found by the writer to be 270 lb for a passive resistance of 750 lb, $r = \frac{750}{270} = 2.78$. Substituting this value in Equation (83), d is found to be $0.705\,t$, or 8.45 ft.

As previously found, the sum of the active pressures is equal to 24 520 lb; the sum of their moments about Point T equals 448 000 ft-lb; and the sum of their moments about Point B equals 215 000 ft-lb. With GS_2, Fig. 18(b), varying at the rate of 750 lb per ft of depth, and H_3 representing the pressure, $J_2\,N_2\,S_2$, that will have to be deducted in order to balance moments about the point, T, then, $H_3\left(27 - \frac{8.45}{3}\right) = (2.78 - 1) \times 448\,000 = 800\,000 = 24.18\,H_3$, and, $H_3 = 33\,100$ lb; from which $\overline{J_2 S_2} = \frac{33\,100 \times 2}{8.45} = 7\,830$ lb.

As $\overline{BS_2} = 750 \times 12 = 9\,000$ lb, $\overline{BJ_2} = 1\,170$ lb. Furthermore, the value of A_o, for this condition, was found to be 3 420 lb.

The resulting pressure distribution, after subtracting the active pressure from the passive resistance, is $\overline{TEC_2} + \overline{C_2\,P_2\,U_2} + \overline{U_2\,R_2\,B}$, as shown on Fig. 18($b$). The author's pressure line, from Fig. 14, is reproduced as a broken line. This pressure distribution is higher and also comes to the left of that just described. This is due to the fact that, in accordance with Mr. Baumann's analysis, as shown in Fig. 11, the passive resistance line is a convex curve instead of having a linear variation.

To approach more nearly the author's resulting pressure line, and still assume a linear variation for the passive resistance, a resistance factor of 3 was assumed, or a value of 900 lb per sq ft. On this basis, $r = \frac{900}{270} = 3.33$, and $d = 8.6$ ft, giving a resultant pressure distribution, $\overline{TEC_3} + \overline{C_3 P_3 U_3} + \overline{U_3 R_3 B}$. This approaches more closely the resultant pressure distribution shown by the broken line. The value of A_o for this condition was found to be 2 980 lb which compares very closely with Mr. Baumann's value of 2 920 lb. It may be noted that the pressure at the bottom of the sheeting decreases consistently as the value of r increases.

While the author states that the compression on the soil depends on the movement of the sheeting, it appears that he overlooked this point when obtaining the deflection curves on the diagram to the left of Table 2. For the curve marked, $e_p = 1.00$, for instance, the shift at the bottom of the sheeting is indicated to be zero. For such a unit resistance, the passive resistance distribution would be the same as *BGS* in Fig. 18 (*b*), and in order to develop the pressure, *BS*, the sheeting would have to move forward at the bottom.

The method of designing sheeting by considering the resultant pressures—that is, the passive resistance minus the active pressure—involves an element of risk in that the actual pressures may be overlooked. This statement is prompted by the manner in which the author obtains the factor of safety against the wall being pushed out for the case illustrated by Fig. 7. In the text preceding Equation (2), he finds this factor of safety to be 2.42. The writer, however, found this factor of safety to be 1.64 in a manner outlined as follows:

Sum of A_1 to A_9, inclusive (Fig. 7), in pounds........	= 12 155
Active pressure, in pounds, below ground surface $= \frac{14.25}{2}(620 + 480)$..............................	= 7 850
Total active pressure, in pounds......................	= 20 005
Anchor stress, in pounds, is given as..................	= 7 555
Pressure, in pounds, to be resisted at the bottom of the sheeting	= 12 450
Total passive resistance, in pounds, $= 201 \times \frac{(14.25)^2}{2}$	= 20 400
Factor of safety $= \frac{20\,400}{12\,450}$..............................	= 1.64

As indicated, this factor of safety (1.64) was obtained on the basis of the passive resistance being 201 lb, which is twice the theoretical value for this case.

Raymond P. Pennoyer,[22] Assoc. M. Am. Soc. C. E. (by letter).[22a]—An ingenious treatment of an involved subject is described in this paper which clearly indicates careful research, and which contains valuable contributions toward the logical design of sheet-pile bulkheads.

Although the writer questions whether Mr. Baumann's accurate development of the passive resistance of soil can as yet be made of practical value, in view of the present-day uncertain knowledge of the other essential properties, and although he differs very seriously with the conclusion that 75% of the interlocked value of a sheet-pile can be assumed, his paper has shown methods whereby some of the problems encountered in design can be solved along sound engineering lines.

One example that has been of great assistance to the writer is the application of Culmann's *E*-line method to the passive resistance value of a berm.

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[22] Structural Engineer, Carnegie Steel Co., Pittsburgh, Pa.

[22a] Received by the Secretary, June 13, 1934.

It is regrettable that Mr. Baumann did not elaborate his test apparatus, indicated in Fig. 2, and that he did not take more deflection observations under various loadings, because the conclusions reached by his excellent treatment of the theory involved would then have been much more convincing.

A steel frame, as nearly rigid as possible, around the enclosure, with the steel sheet-piles bearing against it, would have eliminated the necessity of assuming the supporting value of the soil, and the problem would have been reduced to a study of a simply supported beam with a fixed support at one end. If, after observing deflections, the frame had been removed, the supporting value of the soil could then have been determined accurately from another set of loads and deflection readings.

To add to the uncertain value of the test, only one set of deflection readings was secured, and that only after the steel was stressed beyond its elastic limit. This is clearly indicated in Fig. 6, showing the permanent "set" of the steel after the test. Mr. Baumann does not state whether the measurements were taken before or after the sheet-piles were pulled, or whether the piles were examined for straightness before the test. The fact that they all showed different amounts of "set," illustrates the erratic behavior of steel when stressed beyond its elastic limit.

If several sets of deflection readings had been taken with the water at increasing elevations within the enclosure, so that the steel was loaded within its elastic limit, and if the results were then examined, a conclusive study would have resulted. The writer, however, ventures a statement that, as the load increased on the sheet-piling and slippage progressed in the interlocks, values of the interlocked section modulus under the different loads would have been at such variance that no conclusions as to what could actually be assumed for the interlocked value would be possible.

Mr. Baumann is to be complimented on the ingenious and unique procedure whereby he has determined the actual section modulus of the bulkhead from the observed and calculated deflections. However, if the elastic limit of the steel had not been exceeded, the same results could have been secured by a simple calculation, using the known value of 29×10^6 for the modulus of elasticity of steel. The author's attempt to plot a curve giving values for the modulus of elasticity of steel, indicated in his Figs. 4 and 9, as a result of only seven tests, renders the conclusions of doubtful value. Another serious fact that has been overlooked is that steel follows no constant law for the ratio of stress to strain beyond the elastic limit, and the fundamental calculations in the paper are based on steel always having the same strain at the same stress. Steel, when stressed beyond its elastic limit, is most erratic in its behavior. That tests made from similar steels do not then show the same strain for equal stress, is well illustrated in the Final Report[23] of the Special Committee on Steel Columns and Struts. The modulus of elasticity of the steel in the author's Fig. 4 leaves the straight line at about 37 000 lb per sq in. If Fig. 4 were constructed to a larger scale beyond this point, the

[23] *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–1920), p. 1630, Pl. XXXV,

writer believes it would be seen that the curve shown could not have been drawn from accurate application of test values.

The writer further questions the value of the final figures in view of the accuracy that is necessary in the intermediate calculations when these values are founded on graphical readings and interpolations, with special reference to a variable modulus of elasticity of steel which has been assigned a value to an accuracy of 10 000 units by graphics.

The actual longitudinal shear along the interlocks of the sheet-piles in the test at the tie-rod, where it is maximum, can be calculated from the following equation:

$$s = \frac{S\,M_s}{I} \quad \text{.........................} (84)$$

in which, s = longitudinal shear, in pounds per linear inch; S = horizontal shear, in pounds, = 2 920 lb per ft (Fig. 14 of the paper) = 3 833 lb for a $15\frac{3}{4}$-in width of pile; M_s = moment of resistance per foot of width = area (square inches) × distance from center of interlock to the neutral axis of the sheet-pile = 7.9 × 1.7 = 13.43; and, I = interlocked moment of inertia = 27.35 in.^{3}

The actual shear is found to be about 1 880 lb per lin in. This shear must be resisted by friction at the point investigated because, if the shear exceeds the friction value at any point, slippage will occur. Any claim that a sheet-pile interlock sufficiently free to drive, no matter what its design, will, after driving (and without welding or otherwise positively preventing slippage), develop sufficient friction in itself to eliminate slippage when the load is applied, cannot be substantiated.

From a study of the figures derived, it is difficult for the writer to understand what has justified Mr. Baumann's conclusion that 75% of the full interlocked value of a sheet-pile can be assumed invariably. The only certain value which can be assigned to a sheet-pile in which the interlocks are on what must be the neutral axis in assuming an interlocked value, is that of the single, non-interlocked pile, when one considers such variables as conditions of loading, straightness, lengths of the sheet-piles, and unavoidable irregularities in the interlocks. The sheet-piles might even hold to one value during the short period of a test, but there is no guaranty that the interlocks will not slip later and that the value will be reduced to that which approximates the non-interlocked single pile. The interlocked value, or any portion thereof, should not be used, unless the interlocks are welded or longitudinal slippage is positively prevented in some other manner. There is danger of overstressing the steel in assuming a section modulus greater than that of the sheet-piling.

The writer emphatically denies that American manufacturers advocate the use of 75% of the interlocked value of a sheet-pile, as stated in Conclusion 5 of the paper. On the contrary, American manufacturers have always been consistent in advocating only the use of the single, non-interlocked value of a sheet-pile, unless the possibility of longitudinal slippage is positively eliminated.

Penetration.—The author does not explain the derivation of Equation (3) for the calculation of the penetration of the piling which develops the full constraint of the earth. This equation may be compared with a similar one derived by Dr. H. Blum and the writer in another connection.[24] Its derivation is here illustrated in Fig. 19(*a*). The passive resistance of the

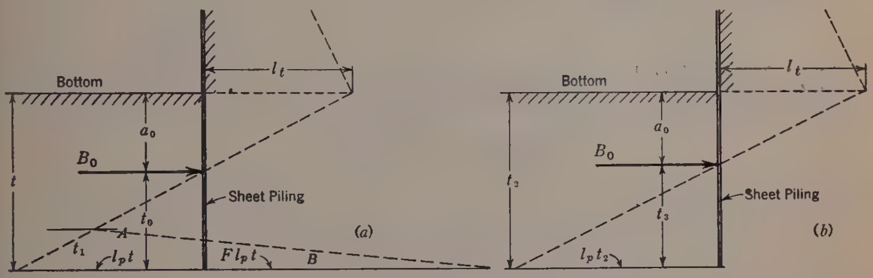

FIG. 19.

soil is treated as linear. The method eliminates any necessity for approximating the relative passive value of the soil on the front and the back of the wall.

With Mr. Baumann's notation let: $\lambda_p - \lambda_a$ = net increment of the passive resistance value of the earth, as developed by the Coulomb formula; e_t = active pressure at the bottom, in pounds per square foot; and B_0 = equivalent reaction of the equivalent simple beam on the bottom support, in pounds per foot of width as determined graphically.

Doubling the value of the passive resistance as derived by the Coulomb formula, the location of the support of the equivalent beam below the bottom is, in feet:

$$a_0 = \frac{e_t}{2\,(\lambda_p - \lambda_a)} \quad \text{.......................}(85)$$

Furthermore,

$$\text{Area } A = \frac{(2\,\lambda_p - \lambda_a)\,t_0^2}{2} \quad \text{.....................}(86)$$

and,

$$\text{Area } B = \frac{(2\,\lambda_p - \lambda_a)\,t_0\,(I + F)\,t_1}{2} \quad \text{..................}(87)$$

in which, F = the ratio of the passive resistance value of the soil on the right or fill side of the wall to the left or outside. Equating pressure loads

$$B_0 = \frac{(2\,\lambda_p - \lambda_a)\,t_0^2}{2} - \left[\frac{(2\,\lambda_p - \lambda_a)\,t_0\,(I + F)\,t_1}{2}\right] \quad \text{.......}(88)$$

and,

$$t_1 = \frac{(2\,\lambda_p - \lambda_a)\,t_0^2 - 2\,B_0}{(2\,\lambda_p - \lambda_a)\,t_0\,(I + F)} \quad \text{....................}(89)$$

[24] *Civil Engineering*, November, 1934, Equation (9), p. 618.

Equating moments around the bottom of the pile:

$$B_0\, t_0 - \left[\frac{(2\,\lambda_p - \lambda_a)\, t^3_0}{6} + \frac{(2\,\lambda_p - \lambda_a)\, t_0\,(I + F)\,(t^2_1)}{6}\right] = 0 \quad (90)$$

Substituting the value of t_1 from Equation (89):

$$t^4_0 - \left[\frac{B_0\, t^2_0}{2\,\lambda_p - \lambda_a}\right]\left[\frac{6\,(I + F) - 4}{F}\right] - \frac{4\,B^2_0}{(2\,\lambda_p - \lambda_a)^2\,F} = 0 \quad (91)$$

For simplification, the loading as illustrated in Fig. 19(a) can be approximated as in Fig. 19(b). Equating moments:

$$B_0\, t_3 = \frac{(2\,\lambda_p - \lambda_a)\, t^3_3}{6}$$

and,

$$t_3 = \sqrt{\frac{6\,B_0}{2\,\lambda_p - \lambda_a}} \quad (92)$$

Let the ratio of t_0 (Fig. 19(a)) to t_3 (Fig. 19(b)), equal a factor, $K_2 = \frac{t_0}{t_3}$.

Then,

$$t_0 = K_2\sqrt{\frac{6\,B_0}{2\,\lambda_p - \lambda_a}} \quad (93)$$

and Equation (91) can be written,

$$K^4_2 - K^2_2\left[\frac{1 + 3\,F}{3\,F}\right] - \frac{1}{9\,F} = 0 \quad (94)$$

For any value of F (Fig. 19(a)), the value of $K_2 = \frac{t_0}{t_3}$ can be calculated from Equation (94).

Equation (94) indicates that the ratio, K_2, is entirely independent of the soil conditions, including the magnitude of B_0. Furthermore, by increasing the ratio of F, the ratio, K_2, approaches unity. Since, even under the most unfavorable circumstances, the value of F can never be less than 1.0, the depth to drive the piling will never be more than 1.2 times that calculated from the approximate loading condition, as illustrated in Fig. 19(b). Normally, the factor, 1.1, applies, but if the soil confined behind the steel sheet-pile is heavy fill, with a high active lateral pressure and a low passive value, this factor may rise to 1.2.

The final result for the depth to drive the sheet-piling in order to utilize the full constraining action of the earth from Equation (92) is:

$$t = K_2\left[a_0 + \sqrt{\frac{6\,B_0}{2\,\lambda_p - \lambda_a}}\right] \quad (95)$$

Equation (95) corresponds to Equation (3). Using $h_0 = 15$; $\lambda_p - \lambda_a = 525$ lb per sq ft (see Fig. 5); $a_0 = 1.8$ ft; $B_0 = 5\,570$ lb; and $K_2 = 1.2$; the penetration, t, is found to be 10.9 ft by Equation (95), and 10.6 ft by Equation (3). The results are equal for all practical purposes. A useful check

on Equations (84) to (95) is illustrated in Fig. 6 of the paper, indicating that constraint was actually present at 10 to 11 ft of penetration.

Mr. Baumann's analysis of the theoretical distribution of the passive resistance of the soil and its application to the test is most unique, interesting, and thorough. The writer extends congratulations on bringing this study to its probable ultimate of mathematical accuracy. However, judgment is reserved as to the feasible practical application to the wide variety of conditions found in practice.

It has long been recognized that the supporting resistance of the earth was not represented by a straight line, but was nearer that developed by Mr. Baumann. However, the straight-line method of calculation, as used in Figs. 5, 7, and 19(a), will continue to be sufficiently accurate at least until the greater inaccuracies of the basic earth-load assumptions are removed.

Three factors are necessary in Equation (67), all of which depend on the character of the soil, and must be determined by test; and, furthermore, such factors might be variable even in closely adjacent locations. The writer is not prepared to state whether such tests can be made of practical value when it comes to the study of stratified, undisturbed, compacted, natural soil at considerable depths.

He is not convinced that the passive value of a soil will increase continuously, no matter to what extent lateral deflections or movement proceeds, as is inferred in Fig. 15 and in the paragraph containing Equations (61) and (62). Compression of the soil, when the sheet-piling is driven, very probably causes sufficient lateral movement to develop as much passive resistance as is safe to assume. Application of the theory is further complicated when earth of more than one type, or a combination of earth and water, is confined behind the bulkhead, or when the sheet-piling is driven into soils having strata of different values.

Therefore, the writer recommends the use of the straight-line procedure for the calculation of the penetration and the corresponding bending moment. This method more closely approximates the true constraining action of the earth at the bottom of a sheet-pile than it is possible at present to approximate the active lateral earth pressures on a sheet-pile.

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DISCUSSIONS

A GENERALIZED DEFLECTION THEORY FOR SUSPENSION BRIDGES

Discussion

BY MESSRS. JONATHAN JONES, A. MÜLLENHOFF, H. CECIL BOOTH, JACOB FELD, AND GLENN B. WOODRUFF, HOWARD C. WOOD, AND RALPH A. TUDOR

JONATHAN JONES,[7] M. AM. SOC. C. E. (by letter)[7a].—Mathematical tools are provided, in this valuable paper, for more accurate study of suspension bridge economy in two categories—the span of less than 1 000 ft, and the continuous span with tie-cables—both of which, from present indications of the trend, should engage the attention of many highway bridge engineers in the United States. The author has been able to present an orderly process for making the computations, as well as a development of the theory, in a compact and usable form.

Two of the practical conclusions in the paper seem to warrant comment:

(1) In the "Introduction," the statement is made that the continuous truss offers advantages "in improved and simplified supporting details at the towers." This is true when the tower legs are battered so that the trusses pass between the legs and rest on a transverse girder. In that case one bearing per truss, instead of two, will be more simply accommodated. For towers with vertical legs in the truss planes, the opposite is true; it is not so simple to pass a truss through a tower leg, at a point of portal bending stress, as to pin it off front and back. However, as the author indicates that the probable field for the continuous truss is in shorter spans, to which past practice also indicates that the battered tower is appropriate, the statement here criticized will hold in general.

(2) In the "Comparison of Deflections," there occurs an averaging of main-span and side-span deflections, which seems to fit no conception that the writer can appreciate. The standard conception is that a deflection, occurring over a certain length of span, represents a change in rate of grade which may or

NOTE.—The paper by D. B. Steinman, M. Am. Soc. C. E., was published in March, 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: In May, 1934, by E. Pavlo, Esq.

[7] Chf. Engr., McClintic-Marshall Corporation, Bethlehem, Pa.

[7a] Received by the Secretary May 10, 1934.

may not be unsatisfactory for the conditions. Now it may happen that the main-span deflection in a given bridge is the criterion of unsatisfactoriness or, in some other bridge, the side-span deflection. An unsatisfactory main-span deflection, however, is not made satisfactory by the circumstance that the side-span deflection is moderate, nor *vice versa*. Therefore, it appears to the writer that the author's examples have shown the 800-ft continuous truss to be slightly more flexible in the main span and considerably less flexible in the side spans than the discontinuous truss of the same economy, and that there the comparison rests. Similarly, in the second study, the continuous truss which costs 5% less than the discontinuous, is a truss 5.4% more flexible in the main span and 6% less flexible in the side spans, whichever of those two percentages may be significant.

Economic comparisons of suspension bridge types, to be valid, must be based on equality of maximum deflection in that span (main span or side span) where deflection is significant. This is true because two suspension bridges may be the same as to loads and unit stresses (that is, as to safety), and yet be as far apart as the poles in flexibility. Unless they both fall within whatever criterion of flexibility is applicable, while one may be an acceptable bridge, certainly both are not.

The design of other types of bridges is based upon their carrying certain loads at certain unit stresses; but a suspension bridge design cannot even be begun until there are stipulated, in addition to the loads and unit stresses, the criteria for maximum admissible flexibility. To a certain extent these criteria must be psychological, possibly as much so as mechanical. Whether such criteria of flexibility as have been and are being chosen are valid, and to what extent deviations therefrom would be regretted, is the most important remaining question in the field of suspension bridge design.

A. MÜLLENHOFF,[8] ESQ. (by letter)[8a].—The theory presented in this paper is as complete and simple as is possible in dealing with such an intricate subject as the exact theory of suspension bridges. Under "Introduction," the author gives a short history of the exact theory and states that it was originated by J. Melan. From the bibliography in Melan's work cited by Mr. Steinman, credit in turn may be traced to Müller-Breslau. In his paper[9] for the first time Müller-Breslau presents the fundamental differential equation:

$$\frac{d^2\,\eta}{dx^2} - c^2\,\eta = c^2\,f\,(x) \quad \text{.....................(58)}$$

and its solution,

$$\eta = C_1\,e^{cx} + C_2\,e^{-cx} - f\,(x) - \frac{f''\,(x)}{c^2} \quad \text{..............(59)}$$

in which the notation is that of Mr. Steinman's paper. At that time, Müller-Breslau had not developed formulas for use in computations by that method and, evidently, he did not then recognize the fundamental difference and the

[8] Prof., Technische Hochschule, Aachen, Germany.

[8a] Received by the Secretary May 16, 1934.

[9] "Theorie der durch einen Balken versteiften Kette," by Müller-Breslau, *Zeitschrift* d. Architektur und Ingenieur-Vereins zu Hannover, 1881, p. 76.

superior value of his solution over the ordinary method of computation. Nevertheless, this paragraph in his paper seems to be the first reference to the deflection or exact theory and, therefore, Müller-Breslau should not be omitted from any historical reference to this subject.

The introduction of the function, d, in the paper seems to be a great improvement (see Equations (14) and (15)). The profession should be grateful for it, and this discussion is confined to a few suggestions as to what else could be done along this line.

Cantilever stiffening trusses which have been advocated in several instances (as, for example, in Schachenmeyer's design of the Hudson River Bridge),[10] are not covered by the theory of this paper, of course. However, it is improbable that bridges of this type will ever be built.

The author leaves only two questions to be answered: (1) The variation of the coefficients of reduction which need to be applied to the values for H and the moments computed by the elastic theory in order to obtain preliminary values of H and c with varying values of the moment of inertia; and (2) the exact theory applying to the influence of wind loads.

In designing a structure by the exact theory it is advisable first to prepare a preliminary design by the ordinary deflection theory and to reduce the values of H and I (which depend on the moments) by certain percentages, in order to obtain approximate values for the final design. These percentages depend mostly on the rigidity of the stiffening truss, or on its moment of inertia, which can vary between very wide limits from zero to almost any value. The author gives the percentages of reduction for a certain case (see Fig. 3), but these do not apply for a design with widely different proportions. It seems desirable to arrange these reduction factors graphically or by formula in such a way that they can be selected and applied to every case.

Perhaps it would be possible to determine this relation either in a general way or by solving a number of cases. The results could be plotted in a graph using as the argument, $\rho = \frac{M_o}{M}$, in which, M is the moment that the stiffening truss is capable of resisting, and M_o, the moment producible by the live load in a beam of the same span length, without cable suspension. Under "General Data—Calculation of Constants," the author uses the following data: $l = 800$ ft; $l_1 = 400$ ft; $p = 1\,300$ lb per ft; and $I = 1\,960$ in.²–ft.² The depth of the truss, h', is given as 14 ft. Then, if the allowable stress is, say, 20 000 lb per sq in., $M = \frac{2 \times 1\,960 \times 20\,000}{14} = 5.6 \times 10^6$ ft–lb.

A few trials will be sufficient to decide whether M_o may be taken as the moment of a simple beam the length of the main span (that is, $\frac{1\,300 \times 800^2}{8} = 104 \times 10^6$ ft–lb), or whether it is necessary to use the maximum or the minimum moment in a continuous beam of the same dimensions.

[10] *Transactions*, Am. Soc. C. E., Vol. 97 (1933), p. 36, Fig. 19(*b*).

In Fig. 6, for example, the maximum moment, M_m, equals 52×10^6 ft-lb, and the minimum moment, M_I, equals -62×10^6 ft–lb. For the first case, therefore, $\rho = \frac{104}{5.6} = 18.6$, and for the second case, $\rho = \frac{62}{5.6} = 11$; or, $\rho = \frac{52}{5.6} = 9.3$. The advantage of this basis of comparison would be its independence from the values of l, I, and h', since ρ is a dimensionless number.

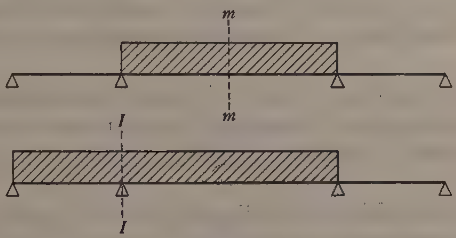

FIG. 6.

Another phase of the problem that has not yet been answered is the influence of the wind. Leon S. Moisseiff, M. Am. Soc. C. E., and Mr. Frederick Lienhard have introduced a good means of approximating this effect.[11] Messrs. Moisseiff and Lienhard, however, do not consider the interdependence between the stresses in the cables and stiffening trusses and the stress in the wind girder. Professor Domke has contributed[12] somewhat to this subject. He treats only single-span bridges, using the elastic method, of course. The stress in the chords of the lateral (wind) truss are compression on the windward side and tension on the lee side. Ordinarily, the chords of the wind truss also serve as the chords of the stiffening truss; and, therefore, the compressive stress will cause an upward deflection which will diminish the pull in the cable on that side, whereas the tension will produce a downward deflection which will increase the pull in the cable on that side. In the one case, therefore, the value of H will be less than by the ordinary method of computation, and, in the other case, it will be greater unless the aforementioned interdependence is taken into consideration. (The writer assumes the wind truss to be in the plane of the lower chords.) On the other hand, a non-symmetric loading on the bridge will produce further stresses in the chords which, in turn, will produce additional stresses in the wind truss. Professor Domke gives an example in which the maximum shear in the 400-ft wind truss is shown to be 22% greater than is indicated by the customary manner of computing the stresses.

Instead of using the two cable pulls, H_1 and H_2, as redundant forces, Professor Domke has found it advantageous to use $X_a = H_1 + H_2$ and $X_b = H_1 - H_2$, because then $\Delta_{ab} = 0$. A more exact theory than that of Mr.

[11] *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 1080.

[12] "Ueber den Windverband versteifter Hängebrücken," by Prof. Domke, in "Festschrift Müller-Breslau," pub. by A. Kröner, Leipzig, Germany.

Moisseiff, termed "the elastic distribution method," seems desirable because for long bridges the influence of the wind load is relatively great. What is the justification, for example, of computing stresses in a bridge exactly within, say, 5% when the stress from the wind may be in error as much as 20 per cent.?

In closing, the writer wishes to record his most sincere respect for Mr. Steinman's valuable paper.

H. CECIL BOOTH,[13] ESQ. (by letter).[13a]—With the present paper by Mr. Steinman and the work of the late Max Am Ende,[14] the theory of suspension bridges seems to have been so thoroughly investigated as to leave little more to be added.

The entire question of the design of a suspension bridge for a particular case depends primarily upon a practical determination of the deck deflection that is to be permitted. Economy in design is dependent upon the extent of this deflection.

No writer on this subject, including the author, seems to have given an opinion as to the maximum allowable deck deflection (say, in terms of deck deflection to span) for suspension bridges of varying spans and for different classes of traffic; and it would be very useful if the author would establish his recommendations.

JACOB FELD,[15] ASSOC. M. AM. SOC. C. E. (by letter).[15a]—The assumptions from which the basic formulas of this paper are derived, are clearly stated and the limitations of each resulting formula are carefully explained. Although there is a great deal of summarized mathematics, the logical steps required for the derivation of each formula can easily be filled in by any one so inclined.

The determination of stresses in structures that are classified as rigid, in which changes of shape are caused entirely by the change in lengths of members due to elastic deformation, for the determination of principal stresses, requires the rules of statics only. However, when structures include cables for total or partial support, the geometrical shape of the cable is quite important in determining the stress, not only in the cable but in all the rigid members. The inter-relationship of the stresses in the cable and those in the stiffening truss members are clearly stated by the author. It is also pointed out that the geometrical shape of the cable is closely related to that of the stiffening truss. However, to avoid unwieldy complications for the formulas, it is assumed that the main cable remains a parabola under all conditions of dead and live load. A weightless cable assumes the shape of a parabola when it is acted upon by a series of equal loads, equally spaced. In a suspension bridge these loads, of course, are the suspender loads. As

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[13a] Received by the Secretary June 1, 1934.

[14] "Suspension Bridges with Stiffening Girders," by Max Am Ende. *Minutes of Proceedings*, Inst. C. E., Vol. CXXXVII (1898–99).

[15] Cons. Engr., New York, N. Y.

[15a] Received by the Secretary May 31, 1934.

indicated by Equation (11), the deflection theory shows that the suspender loadings are not constant, but depend on deflections in the stiffening truss. This is especially true in short spans where, in addition, the ratio of live load to dead load is much greater than in long spans. The inequality of suspender loads will necessarily change the shape of curve in the cable. Such change involves a revision of the cable tension as well as a change in all stresses that depend on the cable tension for their valuation.

In addition, especially for short spans, the condition of unbalanced live load, such as that shown in Cases III and IV can only result in a curve that is not a symmetrical parabola as originally assumed, but that can be taken approximately as two half parabolas, of different characteristics. The lowest point of the curve will not be at the center line of the span.

For long-span suspension bridges, the aforementioned refinements are negligible. However, for short-span bridges, if the ratio of live load to dead load approximates unity, a noticeable change in shape of curve can be determined. The amount of change will increase with an increase in ratio of sag to span, and for the shorter spans the sag ratio is usually greater than for the long spans. Such increase of sag ratio tends to produce a better looking bridge for short spans.

The profession is greatly indebted to the author for such a complete derivation of formulas for all possible cases of suspension bridges in accordance with the deflection theory. With this study available, there is probably no further reason for using the more approximate elastic theory and, in that replacement, this paper serves as a milestone in the advancement of engineering as a science. It could well serve as a model for future reports on the theoretical developments in any branch of engineering.

GLENN B. WOODRUFF,[16] M. AM. SOC. C. E., HOWARD C. WOOD,[17] ASSOC. M. AM. SOC. C. E., AND RALPH A. TUDOR,[18] ASSOC. M. AM. SOC. C. E. (by letter).[18a]—A commendable contribution to the theory of suspension-bridge analysis has been made by Mr. Steinman, and his paper merits a most careful study. In particular, his extension of the "Deflection Theory" to include (1) the effects of stiffening trusses continuous through the towers; and (2) multiple-span bridges with and without auxiliary tie cables, gives the designing engineer two new tools for which there may well be a growing need in the future.

As pointed out in the paper, continuous trusses have already been used on several structures. The author indicates the advantages that this feature possesses, but it is believed that he has given too little emphasis to the added rigidity contributed.

In the usual layout the maximum roadway grade is to be found on the side spans. Live loads produce a change in the grade which is often of

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[18a] Received by the Secretary, June 22, 1934.

serious proportions. A continuous truss will reduce this change materially and particularly so on the side spans. Furthermore, continuity of the truss will forestall any abrupt break in the roadway grade.

Probably the most important advantage to be derived from this feature is the reduction of the maximum wind stresses in the truss. This fact is brought out by Mr. Steinman, but is of such importance as to bear repetition.

Together with the advantages to be derived from continuity, there are certain practical considerations that must not be overlooked. A suspension bridge is somewhat peculiar in that it suffers relatively large distortions. Because of this, the expansion joints at the truss ends must be built to accommodate movements much greater than for temperature alone. Hence, it is not unlikely that for the longer suspension bridges a continuous truss introduces expansion difficulties that cannot be met reasonably. This problem becomes increasingly serious for multiple spans. A combination of a truss continuous through all towers and multiple-span construction is not probable.

Multiple Spans.—The use of a multiple-span suspension bridge has been generally condemned by engineers on the grounds that such a structure would be too flexible. However, there are several possible means of increasing the rigidity, and some are sufficiently meritorious to bear serious consideration:

(*a*) The dead load of the structure may be increased to such an extent that the ratio of dead load to live load becomes larger and the effect of live load becomes small. This would be an uneconomical means of stiffening the bridge and is not worthy of further thought.

(*b*) The moment of inertia (vertical) of the stiffening truss may be increased. Since the stiffness of the truss has such a relatively small effect on the deflections of a suspension bridge, it would be necessary to increase the moment of inertia by such an amount as to make the economics of this method extremely doubtful.

(*c*) The sag ratio of the main supporting cable may be reduced. The stiffness is increased by this means approximately in proportion to the decrease in sag, while the size of the cable must be increased at about the same rate. Thus, if it is desired to reduce the deflections by one-half, it becomes necessary to double approximately the size of the cable. This is a method that may prove the most satisfactory for a given set of conditions when economic, construction and all other phases of the problem are considered.

(*d*) Some system of guys may be used to reduce the movement of the tower tops. An auxiliary "tie cable" running between the tops of successive towers and extending from anchorage to anchorage is such a system. By varying the size and sag ratio of this tie-cable the deflections of a multiple-span bridge can be reduced effectively. Economically, this will compare favorably with any of the other methods.

(*e*) Due to very heavy live loads or to other elements contributing to the flexibility of the structure, it may be advisable to introduce cable anchorage in an intermediate position. This essentially produces independent suspension bridges placed end to end. The west channel structure of the San Francisco-

Oakland Bay Bridge has been so designed. The 900-ft unloaded back-stay, beyond the west side span, together with a relatively heavy live load, made this type of construction advisable.

Incident to the investigation of various multiple-span suspension bridge combinations for the west channel crossing of the San Francisco-Oakland Bay Bridge, means were developed for the necessary calculations. These investigations resolved themselves into two quite distinct phases, the first of which was the preliminary work involving the determination of general dimensions, cable sizes, and distortions. This was followed by an exact analysis of the combination selected to determine moments and shears in the stiffening truss, and other items not obtainable during the first phase.

General Dimensions, Cable Sizes, and Distortions.—For this first phase of the work a modification of the Dana method[19] was developed. This device is only applicable for loadings that are entirely continuous over one or more spans. Since maximum tower and truss deflections and maximum cable stresses are produced by such loadings, these items can be readily obtained. The advantages of the method lie in its relative simplicity, its flexibility, and its accuracy. It involves no calculus. It can be readily made to accommodate any changes in dimensions, loads, or other span constants, and it is particularly well adapted to investigations of multiple spans either with or without tie cables. Finally, it has been found by comparison with model tests to give more accurate values for deflections than those obtained by the "deflection theory." The error in the latter theory is doubtless due largely to the neglect of the change in span lengths.

Final Moments and Shears.—After the establishment of dimensions of the bridge, an exact analysis of the structure (the second phase) to determine the remaining items, was pursued. This investigation was based on equations similar in nature to those presented by Mr. Steinman, the only essential difference being in the treatment of the tie-cable factor. Equation (46) is based on the geometry of the cable. The writers equated the internal work,

$$\left(\frac{H_T}{2} + H_o\right)\left(\frac{H_T L_{sT}}{A_T E_T} \pm w\, t\, L_{TT}\right),$$

to the external work,

$$\left(\frac{H_T}{2} + H_o\right) \Delta l + w_T \int_0^l \eta \, dx$$

The second term of this expression represents the work accomplished in moving the dead load of the tie-cable (w_T) up or down to its new location. The value of η at the center of the span is equal to the change in tie-cable sag, which is expressed as Δf_T. From the equation of the parabola,

$$\Delta f_T = \frac{w_T l^2}{8}\left(\frac{1}{H_o + H_T} - \frac{1}{H_o}\right) = -\frac{w_T l^2 H_T}{8 H_o (H_o + H_T)} \quad \ldots\ldots (60)$$

19 "Tests on Structural Models of Proposed San Francisco-Oakland Suspension Bridge," Appendix *B*. Univ. of California *Publications in Engineering*, Vol. 8, No. 2.

For any other point in the span, η may be expressed in terms of Δf_T, thus:

$$\eta = \frac{4 \Delta f_T \, x \, (l - x)}{l^2} = \frac{w_T \, H_T \, x \, (x - l)}{2 \, H_0 \, (H_0 + H_T)} \qquad (61)$$

The expression for the external work then becomes,

$$\left(\frac{H_T}{2} + H_0\right) \Delta l - \frac{w^2_T \, l^3 \, H_T}{12 \, H_0 \, (H_0 + H_T)}$$
$$= \left(\frac{H_T}{2} + H_0\right) \Delta l - \frac{16 \, f^2_T \, H_0 \, H_T}{3 \, l \, (H_0 + H_T)} \qquad (62)$$

and,

$$\Delta l = \frac{H_T \, L_{sT}}{A_T \, E_T} \pm w \, t \, L_{TT} + \frac{16 \, f^2_T \, H_0 \, H_T}{3 \, l \, (H_0 + H_T) \left(H_0 + \frac{H_T}{2}\right)} \qquad (63)$$

An equation for the main cable was then established in the same manner. The internal work, $\left(\frac{H_1}{2} + H_w\right)\left(\frac{H \, L_s}{A \, E} \pm w \, t \, L_T\right)$, being equated to the external work, $\left(\frac{H}{2} + H_w\right) \Delta l + \left(\frac{H}{2} + H_w\right) \frac{8 f}{l^2} \int_0^l \eta \, dx$. This gave,

$$\Delta l = \frac{H \, L_s}{A \, E} \pm w \, t \, L_T - \frac{8 f}{l^2} \int_0^l \eta \, dx \qquad (64)$$

which is the same as Equation (47) with the terms transposed and expressed in the form, $\Delta l = \frac{H - Z}{Q}$.

The writers anticipated that it would be extremely difficult to solve a single equation for the various simultaneous values of H and H_T. Such a solution would require that these values be first assumed and then the assumptions tested by the equation. In view of this fact, a semi-graphical method for the determination of the simultaneous values of H and H_T was developed and used in lieu of an equation in the form given by Mr. Steinman (Equations (49) and (50)).

This method may be explained best by considering a single span only without reference to the remaining spans of the bridge. In Equation (63) there are only two variables for a given temperature condition. Hence, it is a simple matter to draw a curve expressing the relation between Δl and H_T. In Equation (64) the relation between Δl and H can be established for any given condition of live load and temperature, and a curve drawn. It can be seen that a different curve must be drawn for the main cable for each loading condition.

All curves for one span were drawn on the same sheet. Values of Δl were expressed as abscissa and those of H and H_T as ordinates. Positive values of H were measured up from the origin and of H_T down. In this manner, a vertical scale could be placed on a given value of Δl and the total $H + H_T$

for the span determined by reading the intercept between the proper curves. If, on the other hand, a value of $H + H_T$ was assumed, the corresponding value of Δl could be determined by moving a vertical scale to a position such that it measures an intercept equal to this assumed value of $H + H_T$. The horizontal distance of the scale from the origin measured the value of Δl.

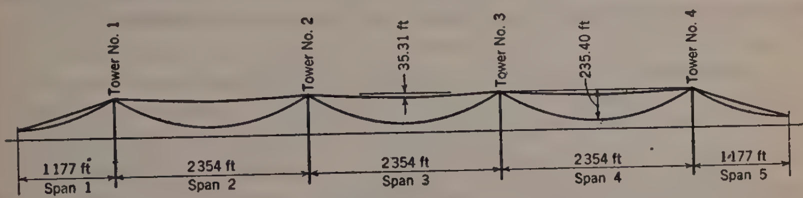

FIG. 7.—GENERAL DIMENSIONS, MULTIPLE-SPAN SUSPENSION BRIDGE.

An outline of the structure, with basic dimensions, is shown in Fig. 7. Other design data are:

Cross-Section Areas, in Square Inches:	
Main cable	494
Tie cable	150
Moment of Inertia, in Inches² Feet²:	
Truss	57 500
Loads, in Pounds per Foot:	
Dead load (excluding tie cable)	8 900
Dead load of tie cable	530
Live load	2 625

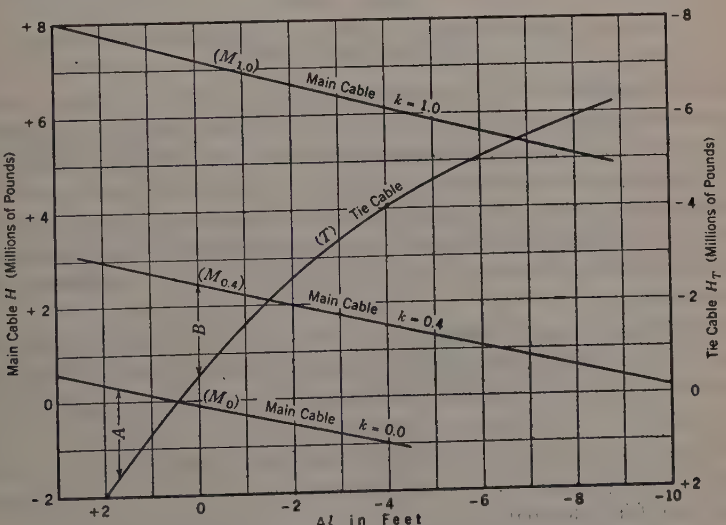

FIG. 8.—H-CURVES FOR SPAN 2, 3, OR 4, FIG. 7 (TEMPERATURE 30 DEGREES FAHRENHEIT ABOVE NORMAL).

In Fig. 8 a set of curves has been drawn for the main spans. These curves include those for the tie cable, and the main cable with no live load on

the structure, with live load covering 0.4 of the span, and with live load covering the entire span. It should be realized that for a complete analysis several other curves for other conditions of live load and temperature would be needed. It would also be necessary to extend some of the curves beyond the limits now shown. In Fig. 9, curves for the side span cables are presented, the main cable curve being for an unloaded condition.

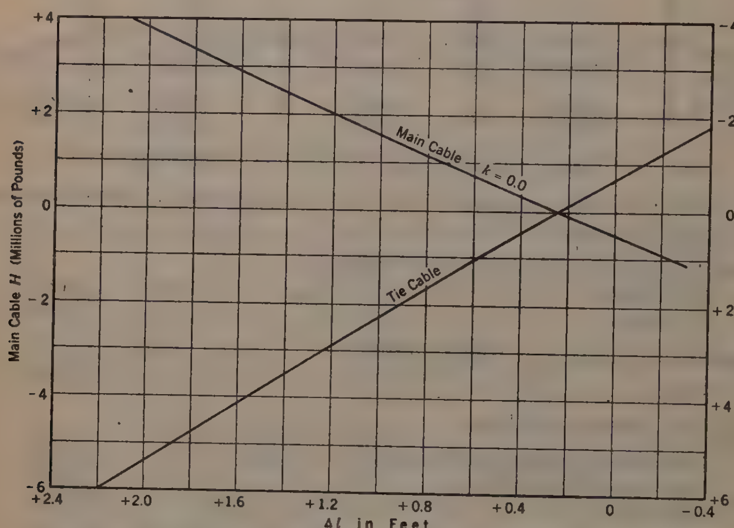

FIG. 9.—*H*-CURVES FOR SPAN 1 OR SPAN 5 (TEMPERATURE 30 DEGREES FAHRENHEIT ABOVE NORMAL).

As an illustration of the use of the curves, assume that there is no live load on a main span and that a value of $H + H_T$ is assumed as + 2 000 000 lb. With a scale, an intercept is measured between the tie-cable curve and the main cable curve for no live load (Fig. 8) equal to this value of $H + H_T$ and is found at *A* (Fig. 8), with the value of Δl of + 1.75 ft. The respective values of *H* and H_T are found by measuring the ordinate to each of the curves at this point. In this case, *H* is found to be + 300 000 lb and H_T, + 1 700 000 lb.

If a live load covering 0.4 of the span is present, the intercept is measured between the tie-cable curve and the main cable curve for this loading. A value of $H + H_T$ of + 2 000 000 lb is located at *B* (Fig. 8), and Δl is found to be + 0.03 ft. *H* is then + 2 460 000 lb and H_T, − 460 000 lb.

A set of curves was drawn for each different span. For the usual conditions with a symmetrical layout, this requires only one set for main spans and another for side spans.

The determination of the several values of *H* and H_T in the various spans was a matter of "cut and try." A value of $H + H_T$ was assumed and the corresponding values of Δl for each span were determined from the curves, as explained. The value of $\Sigma \Delta l$ was then determined for this value

of $H + H_T$, and if it was not equal to zero another value of $H + H_T$ was tried. It was found expedient to solve $\Sigma \Delta l$ for about four assumed values of $H + H_T$ and by means of the plotted relation between the two to determine the satisfying value for $\Sigma \Delta l = 0$. With the proper value of $H + H_T$ determined, the value of H and H_T for each span was readily ascertained from the curves.

It is not difficult to introduce the difference in the total horizontal cable pull between spans due to the resistance of the towers to bending, but normally this can be neglected. Certainly this item will have only a minor effect on truss moments, since the latter are not appreciably affected by variations in H of as much as 10 per cent.

Strictly speaking, this method is not applicable in the case of a bridge with a continuous stiffening truss. However, it appears that, except in the case of short spans with a relatively stiff truss, the effect of continuity may be neglected without introducing serious errors when calculating H-values. The latter could then be used in Equations (9) and (10) to determine moments and shears in the truss. Values of the integration constants must include the continuity factors.

It is significant that a set of curves such as Figs. 8 and 9 can also be used to analyze a bridge without tie cables and of any number of spans. The method could be used readily in conjunction with a structure of the usual type consisting of a single main span.

Practical Considerations.—There are certain difficulties presented by multiple-span suspension bridges with tie cables that should be emphasized. Of first importance is the necessity for providing adequate anchorage between the main and the auxiliary cables at all tower tops. This provision is imperative, since there is a transfer of load from one to the other which runs into millions of pounds on a major structure. The solution of this problem is not impossible, but it is difficult. If the cables are continuous over the towers, some highly effective clamping action must be developed. Discontinuous cables of socketed bridge strands may offer a solution.

It is not difficult to proportion this type of structure so that the vertical deflections of the trusses are relatively small; but it is somewhat of a problem to keep the bending of the towers toward the center of the crossing within reasonable limits. These towers must be strengthened adequately to perform this additional duty. The possibility of hinged towers presents itself, and under favorable conditions, may be the answer.

Several new construction problems would certainly present themselves, but doubtless the ingenuity of American erectors would be fully equal to the occasion.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SAND MIXTURES AND SAND MOVEMENT IN FLUVIAL MODELS

Discussion

BY MESSRS. JOHN LEIGHLY, PAUL W. THOMPSON, AND GERARD H. MATTHES

JOHN LEIGHLY,[28] ESQ. (by letter)[28a].—The experimental data and other evidence presented in this paper serve to establish beyond a reasonable doubt the constancy of critical tractive force for a given sand mixture. The author's use of the du Boys equation in computing tractive force is a great improvement over procedures based on a supposed "bed velocity," and he deserves credit for again setting that venerable equation in a place of honor. Unfortunately, his experiments do not provide the definitive empirical critique of the equation that one would like to see. A possible objection to his method of computing tractive force is that he has used depth rather than hydraulic radius as a measure of the mass of water acting on a unit of bed area. In proceeding thus, however, he was evidently justified by the conditions of his experiments, in which the water had ratios of width to depth between 15:1 and 40:1. Recomputation of the critical tractive force in his observations, using hydraulic radius instead of depth, yields results that do not differ materially from his conclusions.

The ultimate result of Captain Kramer's investigation, and the one to be subjected to the closest scrutiny, is the final equation, which aims to express the mobility of a sand mixture in terms of its mechanical composition. Further work, both experimental and theoretical, must be done before a definitive formulation of this important relation may be had. The author's solution is ingenious; to gain practical ends, Gordian knots must be cut; and in the present instance some quality of the gradation curve is clearly the appropriate weapon. In testing the validity of his equation, Captain Kramer has had to use inhomogeneous data; but lack of homogeneity alone will not account for the wide scattering of plotted points in Fig. 14. It is worth while to examine the predicates of his final equation.

NOTE.—The paper by Hans Kramer, Assoc. M. Am. Soc. C. E., was published in April, 1934, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[28] Asst. Prof. of Geography, Univ. of California, Berkeley, Calif.

[28a] Received by the Secretary May 25, 1934.

The distinctive term, $\frac{1}{M}$, in this equation, is the ratio, $A_A : A_B$, between the areas of the upper and lower divisions of that part of the field of co-ordinates that lies between the axis of ordinates and the cumulative gradation curve in Fig. 12. This ratio is used as a coefficient to the mean particle diameter of a sand, as a measure of the range of size of particle. The precise significance of the numerical value of the coefficient in any given instance is not at all immediately evident. Readers of this paper would be grateful for a description, supported by the citation of a large number of gradation curves, of just what the ratio proves as to the grain distribution of any sand. The following statements do not aim to provide such a discussion, but only to show the effect of applying the ratio to the results of the experimental measurements cited by the author.

Range of grain size is not always, as he implies under "3.—The Ultimate Equation (10)," the primary factor that determines the numerical value of the coefficient, $A_A : A_B$. If one considers gradation curves which (when plotted as are those in Fig. 13) appear as straight lines with their lower extremities at the origin of co-ordinates, the ratio, $A_A : A_B$, regardless of range of grain diameter, will always equal 3. The qualities measured by the departure of the numerical value of the ratio from 3 (in curves of a given range such that their lower extremities are at or near zero of the axis of abscissas) are those that affect the slope of the gradation curve in the several parts of the range: Concentration of grains in the vicinity of a modal diameter; and the location of the modal diameter within the range. The closer the modal diameter lies to the lower extremity of the range, and the greater the concentration of grains about the mode, the higher the value of $A_A : A_B$, and thus, by Captain Kramer's equation, the less the mobility of the sand. That proposition is doubtless worthy of discussion, but the experimental data scarcely suffice to prove or to disprove it. The important fact —a purely geometric one—to be noted is that with the same range of grain size from zero (or very nearly zero) upward, it is possible for the ratio to have any one of a large number of values ranging upward from slightly more than 1, depending in any particular sand on the modal diameter and on the deviations of the diameters of the component grains from the mode. The large possible range of numerical values of the ratio, and the uncertainty of the significance of its variations, make it a dangerous element in any equation designed to express mobility. The danger is always present that it may modify too strongly the essential term of any such equation, the mean diameter.

A second influence exerted on the numerical value of the ratio, $A_A : A_B$, that is independent of range of grain size, may be seen by comparing the gradation curves the lower extremities of which are near zero diameter with those of sands the fine constituents of which have been naturally or artificially removed. The values of the ratio in curves, *B* to *H* and *S* to *V*, in Fig. 13, are determined not so much by the range of grain diameter as by the scalar value of the diameter of the smallest particle in each sand. If a

gradation curve having the slope of Curve T in the diagram be thought of as having a range of diameter from zero to 1.25 mm, the ratio, $A_A : A_B$, will have the numerical value, 3. Let the curve now move in the field of coordinates from left to right, the range remaining constant; as the mean diameter, and, with it, the maximum and minimum diameters, increase, the value of the ratio decreases and approaches 1. That is to imply that range of diameter is important when the smallest grains are very small, but as they become larger the significance of range decreases. That, again, is a proposition that can be discussed, but that rests on shaky evidence.

The objections urged in the foregoing two paragraphs may appear hypothetical; but the experimental data which the author cites show plainly that the use of his ratio actually distorts the fundamental relation obtaining between coarseness of sand and critical tractive force. For example: The mean diameters of the sands represented in Fig. 13 by the curves, J and L, are not greatly different; their ratio is about 1:1.1. The experimentally determined critical tractive forces of the two sands have a ratio of about 1:1.3. The quantities tabulated in Table 3, Column (14), however, that are intended to be proportional to critical tractive force, have a ratio of 1:3.1 for the two sands. This discrepancy is not the result of difference in range of diameter of particle. The ratio, $A_A : A_B$, has, for Curve J, a value of nearly 1.3 and for Curve L, 3.6; but if Curve J were moved to the left in the diagram until the finest constituents of the sand it represents became as fine as the finest in Sand L, the ratio would be 3, thus being nearly as great as for Curve L, although the range would be less than one-half that of Curve L. If the range of Curve J were then increased, the ratio would remain constant at 3, as long as the finest constituent particles had diameters of nearly zero. When Curve J assumed the same range as Curve L, the ratio would still be only five-sixths of the ratio for Curve L. As matters stand in Fig. 13, the diameter of the finest constituents of Sand J, not its total range of diameter, is what determines the numerical value given by evaluating Captain Kramer's final equation, and gives rise to the discrepancy between the computed indices of the critical tractive forces in Sands J and L. Evaluation of the equation without the ratio, or even naive judgment, on the simple basis of relative coarseness, would give results closer to the experimental determinations of critical tractive force than the full equation.

Comparison of the places occupied by Sands C, K, and U in Fig. 14 provides further adverse evidence; the ratio, $A_A : A_B$, gives to these three sands nearly the same index of critical tractive force, although Sands C and U are much coarser than Sand K and although the experimental data show that Sands C and U, as again would be expected from naive consideration of their coarseness as compared with that of Sand K, are far more resistant to traction than Sand K. The ratio separates sands that belong together, and brings together sands that differ widely when judged according to the primary criterion—mean diameter.

Finally, if the values of A in Column (10) Table 3, proportional to the mean diameters of the several sands, are plotted against the experimentally

measured critical tractive forces of Column (3), the correlation, particularly for the coarser sands, is much better than that shown in Fig. 14; and the evidence of the coarser sands should be given more weight than that of the finer ones, since with them the errors of observation do not weigh so heavily in the results of computation.

It is altogether probable that an ideal equation expressing critical tractive force as a function of mechanical composition of sands would contain terms representing qualities other than mean diameter. Certainly, however, the derivation of these terms must begin from the variations seen in the results of experiment when mean diameter is held constant or is eliminated by computation, while range of grain size, modal diameter, and dispersion of grains through the range of diameter are varied. Furthermore, the term or terms included, in order to correct for these minor traits of mechanical composition, will be small as compared with the numerical expression of coarseness, which must remain the fundamental term in any such equation.

PAUL W. THOMPSON,[29] JUN. AM. SOC. C. E. (by letter).[29a] — Among the important yet-to-be-solved problems that confront the small-scale-model experimentalist is that of simulating movement of stream-bed materials. In developing a practicable technique for the attack of this problem, Captain Kramer has performed a service valuable to the science of hydraulic model experimentation. That much of immediate value has resulted from the author's work is evidenced by the fact that his methods and reasoning are now being applied and extended in several of the leading laboratories of the United States. However, so difficult and complex is the problem of simulating the movement of stream-bed materials that despite the author's work the problem still seems all but impossible of solution. So much remains to be known that by comparison almost nothing is known.

Consider the actual bearing which the author's studies have on the problem of simulating movement of stream-bed materials. His experiments have to do with determining the critical tractive forces for certain sands and with analyzing these data to develop a formula for determining critical tractive forces for these and other sands. The "geschiebe" movement limit, quoted by the author, requires that bed-load movement begin and end in the model at stages corresponding to those for like conditions in Nature. If bed movement in the model is to be similar to that in Nature, the definition of this limit should be revised to require that, at all stages, bed-load movement in the model be similar to what it is at corresponding stages in Nature. To satisfy the limit as thus stated necessitates that cognizance be taken not of relative critical tractive forces, but of relative rates of movement of the bed loads. The distinction between critical tractive force and rate of movement is brought out by the author's definitions, enunciated in the case of critical tractive force, but implied only in the case of rate of movement.

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[29] Second Lieut., Corps of Engrs., U. S. Army, Omaha, Nebr.

[29a] Received by the Secretary, June 18, 1934.

Thus, critical tractive force is that lowest value of slope × depth × unit weight of water which results in general movement of the sand in question; rate of movement is simply the total quantity of bed load moved through any cross-section per unit of time.

The important point to be noted is that the relations existing between the critical tractive forces of given sands are not the same as those existing between the rates of movement of those sands. This fact is demonstrated by data derived from experiments conducted at the U. S. Waterways Experiment Station. In another connection, Lieut. K. D. Nichols has given a table of the critical tractive forces for four sands studied in these experiments and a set of curves showing the rates at which the same sands move under varying tractive forces.[30] Although the critical tractive force for Sand *C* of the curves cited is about three times that for Sand *B*, the rate of movement for Sand *C* is much greater than for Sand *B* in most of the cases shown. The opposite of this fact is indicated for Sands *B* and *A*. If the curves for Sands *B* and *C* were continued to the left, they would cross; perhaps if they were continued to the right, they would cross again; and, in any event, it must be concluded that the relations between the critical tractive forces of these sands in no way defines the relations between the rates of movement of the sands. If only data concerning critical tractive forces are known, it is not warrantable to draw therefrom conclusions regarding rates of movement of the sands in question. Therefore, data concerning critical tractive forces throw little light on the matter of the "geschiebe" movement limit in the case where this limit is stated so as to give it practical bearing on the problem of the simulation of movement of stream-bed materials.

The writer does not intend to convey the impression that, given data secured through flume tests on rates of movement of the sands in question, it follows that simulation of bed movement may be obtained. Too many other factors are present to complicate this problem. Until the movement of débris by natural streams is understood thoroughly, it will obviously be impossible to simulate such movement.

The law of the constant critical tractive force (the truth of which is essential to most of the "practical applications" listed by the author) is certainly true for the sands investigated through the ranges of values investigated. However, it is doubtful whether this law remains true for slopes and depths of the order of those in Nature. It is difficult to believe that a given depth-times slope in a model stream which has a value for Reynolds number of, say, 10 000, will move the same limiting mixture of sand as a natural stream that has the same depth-times slope, but which has a value of Reynolds number of, say, 1 000 000. Still more difficult is it to believe that the rate of movement of a sand would be the same under the conditions cited. Movability depends, quantitatively, on the type of movement. With a low value of Reynolds number, saltation would probably be relatively less important than with a high value of the number, and the rate of movement of the sand would certainly be affected thereby. It may be noted that divergencies

[30] *Proceedings*, Am. Soc. C. E., February, 1934, p. 280, Table 4 and Fig. 6.

in the value of Reynolds numbers of the order noted herein actually occur between model and Nature in most cases. This fact is evidenced by a consideration of the data in Lieut. Nichols' Table 4, previously cited.

The conclusions stated under "1.—The Concurrence Limit" must be questioned. It is stated therein that in river models using sand beds both gravity and friction exert determining influences on the phenomena of flow. It is stated that this fact is responsible for the acceptance, qualitatively but not quantitatively, of data secured from a movable bed model. The friction referred to in this limit is the internal friction due to the viscosity of the fluid, while the gravity referred to is the familiar force, $g \times$ weght. It is not apparent how the presence of a movable bed in a model would act to increase the importance of the frictional force over the force of gravity. As far as the relative importance of the frictional and gravity forces goes, open-channel models with fixed beds certainly enjoy no advantage over those with movable beds. However, a "concurrence limit" that does operate to forestall the acceptance quantitatively of data from a movable bed model derives from the fact that quantitative movability of sand defies simulation.

It must be emphasized that Captain Kramer's work is of far-reaching value. He has blazed a trail into fields which heretofore most experimentalists have feared to tread. His success in arriving at ertain rational conclusions has encouraged other experimentalists to undertake the difficult problem of the simulation of the movement of stream-bed materials. Whether or not this problem is ever solved, Captain Kramer will be entitled to rank high among the pioneers who have advanced the science of hydraulic model experimentation in the United States.

GERARD H. MATTHES,[31] M. AM. SOC. C. E. (by letter).[31a]—The relation between the movement of the sedimentary load and the tractive force of flowing water is clearly shown in this paper. It should be illuminating to those members of the profession who are still struggling with the complex relations between transportation of sediment and current velocity. The tractive-force formula omits velocity from consideration and is of a simplicity that merits the attention of all engineers concerned with practical river problems. It applies as much to living streams as it does to laboratory models.

The opposing effects on bed-load movability of grain size and void ratio have always been difficult to visualize. The author's approach to ascertain these effects by following the method used by Hummel[27] in determining the strength of concrete from sieve analysis of the aggregate is ingenious and promising. It is hoped that this method will be further developed by experiments on other sand mixtures to the end that the constant in Equation (10) may be determined definitely.

As pointed out by Captain Kramer, his laboratory determinations of bed-load movement can be applied advantageously to actual river problems. In

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[31] Prin. Engr., Office of Pres., Mississippi River Comm., Vicksburg, Miss.

[31a] Received by the Secretary June 27, 1934.

[27] "Die Auswertung von Siebanalysen und der Abram'sche Feinheitsmodul," von A. Hummel, *Zement*, No. 15, 1930.

1933, the writer undertook to apply the knowledge gained from the research work done in the field of tractive force by the author and others, together with the experimental work at the U. S. Waterways Experiment Station, to Mississippi River problems. The progress thus made in connection with important dredging operations is extemely encouraging, and has demonstrated that, in practice, the slope-depth relation is more satisfactory than any velocity formula. The relation is simple and is not concerned with either time, velocity, or friction factors, being merely a function of mass and gravity. It does not show how much or how fast bed-load materials are moved, but indicates what conditions must exist to keep particles of any given weight and size in motion. It makes clear that a deep river having a light gradient may exert as much tractive force along its bed as a shallow river on a steeper slope, regardless of the velocities in either one. Tractive force may be pictured, non-technically, as the forward sweeping force exerted along the river bottom by the weight of an imaginary column of water (in height equal to the depth of the river), in its movement down stream. This movement is caused by the fall of the river as indicated by its surface slope, and is independent of local inequalities in the river bed. The latter merely serve to shorten or to lengthen the column of water from point to point down stream. It will be seen that tractive force necessarily varies from point to point in any one cross-section and also from point to point down stream.

The writer has found it convenient to use the following method for deriving the du Boys formula, the notations being the same as those used by the author, but with English measures instead of metric. Referring to Fig. 15,

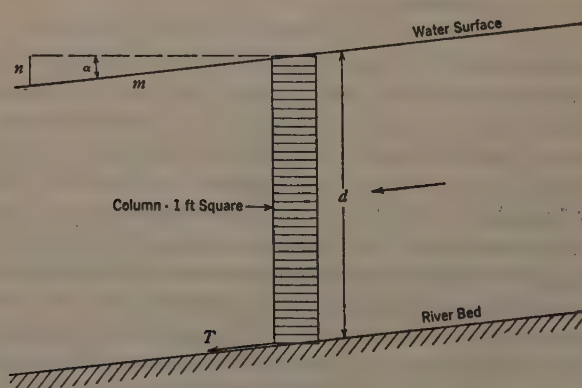

FIG. 15.

let T = the tractive force, in pounds per square foot, of river bed; d = the depth, in feet, equal to the height of a column of water, 1 sq ft in cross-section; ρ = the weight, in pounds, of 1 cu ft. of water; S = the water surface slope $= \frac{n}{m}$; α = the angle of slope; and g = the acceleration due to gravity, in feet per second.

Proceeding from the basic expression; Tractive force = mass × acceleration; mass $= \frac{\text{weight}}{g} = \frac{\rho\, d}{g}$; and, acceleration down an inclined plane $= g \sin \alpha = g \frac{n}{m} = g\, S$. Substituting these values, gives the author's Equation (3). It will be seen that g is canceled, thus eliminating all that relates to velocity and time.

The fact that neither the du Boys formula nor the Kramer formula affords any measure of the rate at which bed material is moved is no limitation to their utilization. In practice, it is seldom important to know how much bed load moves along the river bottom in a given time. The important question is under what conditions movement takes place and, conversely, when movement ceases. After all, the quantity of material of any given kind transported by a natural stream depends on many conditions wholly independent of the available energy for moving such material. This is true of the suspended load as well as of the bed load.

As stated in the paper, flume analysis of the bed-load materials of a stream, whether made in a hydraulic laboratory or in an improvised flume at the project site, will enable an engineer to determine the behavoir of these materials in the stream itself for the prevailing combinations of water surface slope and depth. This enables him to compute readily what minimum depth must be maintained for any given slope, in order that the channel may develop sufficient kinetic energy in the form of tractive effort to transport the heaviest as well as the lightest materials composing the bed load, and thus to prevent shoaling. In its simplest form, the problem, in practice, usually relates to discovering why a river persists in building bars in a particular locality at certain stages, and what should be done to correct this condition.

A study of prevailing slopes and depths, the tractive force values corresponding thereto, and the critical tractive force required to move the various types of bed-load materials, affords the most direct manner of attacking such a problem. In the case of the Lower Mississippi River, such a study has led to the tentative adoption of a value of 0.25 lb per sq ft of river-bed surface as the critical tractive force, T_0, necessary for imparting motion to the entire range of materials predominatingly found on the bed of the river, up to and including loose sub-angular gravel 1 in. in longest diameter, with a specific gravity of 2.65. This value of T_o is predicated on tractive force experiments conducted in a tilting flume at the U. S. Waterways Experiment Station. Table 4 gives combinations of depths and slopes that provide a tractive force equivalent to 0.25 lb per sq ft. Column (1) refers to the actual depth at the point of measurement and not to the average depth or hydraulic radius.

In other words, any reach in the Lower Mississippi River the combinations of depth and slope of which equal or exceed those in Table 4, according to the du Boys formula, should be free from shoaling. The conditions actually found to exist in this river appear to confirm this claim. In the course of the study, extensive sampling of the materials composing the river

bed and of the materials moving by traction was undertaken. Much headway has been made in securing representative samples but, so far, quantitative determination of the amount of bed load in motion, under any given conditions, has met with little success. While much remains to be done toward determining a rational practice on which to predicate plans for channel improvements by this method, the results obtained to date, in general, are satisfactory and appear to confirm the soundness of the slope-depth principle.

TABLE 4.—DEPTHS AND SLOPES NECESSARY TO PROVIDE A TRACTIVE FORCE OF 0.25 POUND PER SQUARE FOOT.

Depth, in feet (1)	Fall, in feet per mile (2)	Slope, S, in feet per foot (3)	Depth, in feet (1)	Fall, in feet per mile (2)	Slope, S, in feet per foot. (3)
25	0.84	0.00016	100	0.21	0.00004
50	0.42	0.00008	125	0.17	0.000032
75	0.28	0.000053	150	0.14	0.000027

The statement by the author to the effect that tractive force applies only to mobile materials, cannot be over-emphasized. Any sedimentary materials that have been submerged long enough to have become compacted, are not readily moved by traction. This is especially true in the streams of the eastern half of the United States where compaction is an important factor, and less so in the streams of the Arid West the beds of which, by reason of being dry many months each year, suffer little compaction. Kinetic energy in forms other than tractive force are required to tear loose compacted sedimentary materials and set them in motion again, whether by traction or in suspended form. Tractive force, therefore, is not synonymous with all forms of erosive energy. However, it is erosive in its action as regards deposits of uncompacted, that is, mobile materials.

It seems incredible that the simple relation between tractive force and slope and depth, should have remained obscure for so long a period, considering that it has been known since 1786. Why so much time and effort have been devoted in trying to link sediment movement with velocity finds its answer in psychology. The forces at work in a natural stream carrying sediment, are not readily analyzed, as the water is too turbid to permit of making eye observations below the surface. The human eye, looking down upon the water surface, is powerless to see such a stream at work along its bottom and banks, the only places where physical changes are wrought; but it is ever beguiled by the ceaseless motion at its surface. This surface motion, both as to velocity and direction, affords no reliable index as to what is going on below the surface (which is all the eye can see). Nothing has been more natural than for Man, through the ages, to try to link all that takes place under a stream surface with the motion visible at the surface, and failing in this, to link it with motion measurable by him somewhere within the liquid prism.

In the last analysis, the movement of water, including that of its sedimentary load, is a matter of mass and energy. Velocity is not a force—it

is not even a good measure of it in the case of flowing water. For this reason it has served as a poor approach to discovering the laws of sediment transportation and deposition. A century and a half of experimentation and research in transportation of débris by running water, in which have participated some of the foremost hydraulic engineers and scientists of the world, has yielded only one definite answer, namely, that there is no simple or direct relation between the velocity in a stream at any point in its cross-section and the movement of its sedimentary load, whether this movement is in suspended form or whether it is sliding, rolling, or saltation. The mathematical relations that have been evolved, whatever scientific merit they may possess, are entirely too cumbersome for use in every-day river engineering problems. Formulas in which bottom velocity is used as the criterion for bed-load movement appear promising, but so far have found little application because the river engineer, in practice, is never in a position to determine bottom velocities and directions of bottom currents quickly or with any degree of reliability. There is no part of any vertical velocity curve that is more difficult of determination than the bottom part, and this is true in a river as well as in a laboratory flume. By contrast, the tractive-force formula has been found exceptionally convenient for practical use. Water-surface slopes may be read from a suitable profile and need be approximate only, while depths may be obtained quickly with a sounding lead to the nearest foot. A simple tabulation such as Table 4, or a diagram, save computation, but even in the absence of such facilities, the computation is readily made in the field.

The paper is fittingly dedicated to the memory of the late Mr. John R. Freeman. From his association with Mr. Freeman in his earnest endeavors to foster hydraulic laboratory practice in the United States, the writer is moved to exclaim: Would that Mr. Freeman could have lived to have seen the fruits of his labors as exemplified by this paper and by the work done at the U. S. Waterways Experiment Station during the two years, 1933 and 1934.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

LABORATORY TESTS OF MULTIPLE-SPAN REINFORCED CONCRETE ARCH BRIDGES

Discussion

BY C. B. MCCULLOUGH, M. AM. SOC. C. E.

C. B. MCCULLOUGH,[4] M. AM. SOC. C. E. (by letter)[4a]—This presentation of laboratory tests of multiple-span reinforced concrete arch bridges is not only logical and coherent, but also gives evidence of an extraordinary degree of care and precision throughout the research. The section entitled "Description of Specimens and Apparatus," which deals with testing apparatus and equipment, has a value that extends beyond the scope of these particular tests, pointing the way to methods for further experimentation along the line of stress distribution in structures that are highly indeterminate and complex. The method for simulating or reproducing foundation fixity at various pier heights is especially ingenious. A similar comment may also be made in reference to the method of measuring rotations.

The scope of the investigation includes three phases of structural analysis concerning which there appears to be a dearth of published data: (1) An experimental verification of the elastic theory as applied to multi-span arches; (2) a determination of the approximate degree to which a variation in the elastic properties of the materials (or a variation in elastic constants due to any other condition) operates to modify stress distribution and to introduce error in the analysis; and (3), the effect of superstructure restraint and a study of the function of expansion joints. All these are questions that have appeared for some time to merit further investigation and study, and the results of Professor Wilson's researches furnish data which hitherto have been needed badly by every engineer dealing with the design of this structural type.

The influence lines indicated in Fig. 8 check the elastic theory with a degree of accuracy that is reassuring, and the two methods, one based upon pier-top displacements and the other on measured reactions, also check in a

NOTE.—The paper by Wilbur M. Wilson, M. Am. Soc. C. E., was presented at the Joint Meeting of the Structural Division, Am. Soc. C. E., and the Applied Mechanics Division, Am. Soc. M. E., Chicago, Ill., June 29, 1933, and was published in April, 1934, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[4] Bridge Engr., State Highway Dept., Salem, Ore.

[4a] Received by the Secretary August 19, 1933.

satisfying manner. The same comment may be made in reference to Figs. 9 and 10 wherein influence lines have been developed by five different methods.

The analysis of a multi-span arch series, utilizing experimentally determined elastic constants, appears to present far-reaching possibilities. Professor Wilson refers to a method developed under his supervision by Donald E. Larson, Jun. Am. Soc. C. E.,[3] in 1930, but does not go into this question in any great detail. The importance of this phase of the research merits more consideration. It is to be hoped that in closing his paper Professor Wilson will find it possible to present this matter more completely.

The entire research appears to be grouped about two major inquiries—the first having to do with the effect of elastic piers in multi-span arch construction, and the second pertaining to the influence of the deck and spandrel structure.

In reference to the former (the effect of elastic piers of varying heights), an inspection of Figs. 13 and 14 discloses the fact that the general effect of increasing the height of elastic piers is to reduce the horizontal thrust, to reduce moments at or near the pier skewbacks, and to increase stress near the center of the span. In this connection an analysis was made in the writer's office a few years ago embracing a three-span arch structure on elastic piers. Because of the certain location studies involved, and as a matter of general interest, analyses were made of three designs, each comprising a series of three 150-ft arches and differing only in the matter of pier heights. Design No. 1 involved a pier height of 45.8 ft; Design No. 2, a pier height of 15.8 ft, and Design No. 3, a pier height of 95.8 ft.

The influence lines for the moment and thrust at three points on Span 1, for each of these three designs, are plotted in Fig 20. These influence lines naturally exhibit the same tendencies as those noted in Professor Wilson's paper; that is, an increase in pier flexibility is accompanied by a decrease in thrust, a decrease in skewback moment at the pier, and a general increase in stress throughout the region adjacent to the crown. It is of interest, however, to note the marked effect of loads in adjacent spans, especially for the two designs involving relatively high piers. Professor Wilson's researches indicate an increase in stress as pier heights are increased, amounting to 13% for the three-span series on 20-ft piers, when compared with a single span of like dimension having fixed ends. When the effect of the deck and spandrel structure is taken into consideration this increase amounts to 17 per cent. A study of the influence lines given in Fig. 20 indicates that a stress increase considerably greater than the foregoing may be expected if the effect of loading in adjacent spans is considered; and, while this fact need cause no serious concern in view of the improbability of simultaneous loading in all spans of a series in such position as to cause maximum stress, such a condition should always be investigated, especially for extreme pier heights.

The second phase of Professor Wilson's paper has to do with the effect of the supported deck and spandrel structure on arch rib strains and stresses.

[3] "A Study of Multiple-Span Arches:" A Thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering in the Graduate School of the Univ. of Illinois, 1930.

In general, his researches disclose a reduction in horizontal thrust and in bending moment at the springing as a result of the placement of an integrally supported spandrel structure. When expansion joints are inserted in the deck, the same tendencies are exhibited, but to a much less degree. In this con-

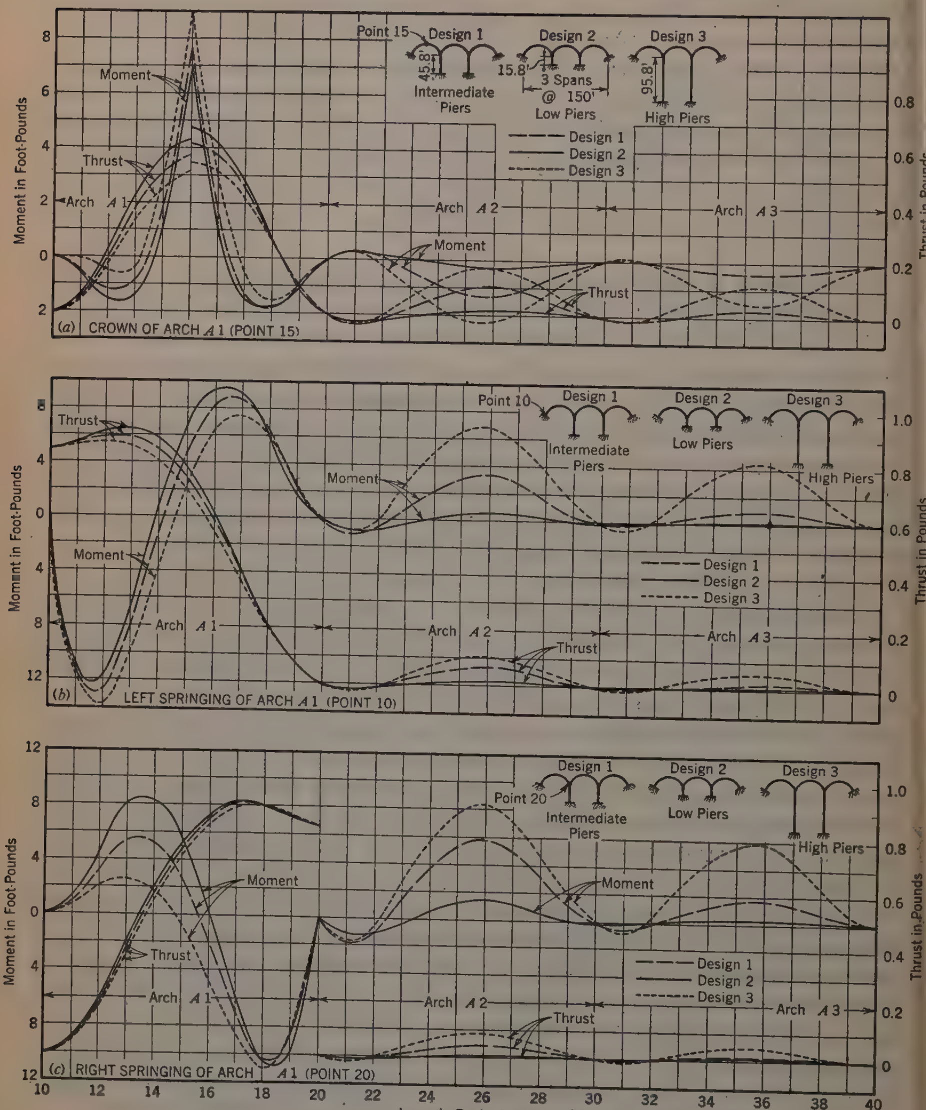

FIG. 20.—COMPARISON OF MOMENTS AND THRUSTS.

nection, certain observations in reference to the effect upon arch rib strains of a spandrel structure and deck, even with the utmost degree of articulation possible, were made by the writer in connection with the operation of decenter-

ing the Rogue River Bridge, in Southern Oregon, in 1932. A full report of a co-operative research on this subject has been published for limited distribution by the U. S. Bureau of Public Roads and the Oregon State Highway Commission.[5]

This bridge was constructed after the Freyssinet system[6] which, as applied in this case, consists in the complete severance of the arch rib at the crown, and the introduction at the severed section of a system of hydraulic jacks by which the arch may be raised from its falsework support and adjusted to any predetermined elastic state. The arch thus handled is converted into two statically determinate cantilever segments, which render it possible to study stress distribution under varying conditions with a considerable degree of accuracy.

The articulation thus attempted[7] was about as thorough as can ordinarily be achieved under construction conditions. In other words, the degree of restraint exercised by the spandrel structure was apparently reduced to as nearly a minimum as is consistent with construction practice. Nevertheless, the observed crown movement was less than one-third of the theoretical for an unrestrained rib. For example, at a total crown-thrust value of 840 000 lb, the computed crown spread for an unrestrained rib amounts to 1.60 in. as against an average observed value of slightly less than 0.50 in.

In connection with the decentering operations, and as part of a general research program inaugurated in connection therewith, electric telemeters of the McCollom-Peters cartridge type[8] were installed, at the extrados and the intrados, at five points along the axis of the arch rib, these telemeters being located at each springing, at each quarter-point, and at or near the crown. The telemeter forces, before decentering pressures had been reached, were found to be much less than the applied or jacking force, indicating that during this stage of the operation a considerable part of the horizontal thrust was being taken by the centering; however, the observations indicated that after the theoretical decentering thrust had been reached, the rib was carrying the full horizontal thrust at the crown section. This is exactly as it should be since the supported spandrel structure could have no effect on crown stress.

Table 5 indicates the percentage of theoretical thrust observed from the telemeters at the various skewbacks and also at the various quarter-points for the second runs. An inspection of these data discloses the fact that the quarter-point stresses are quite uniform. In each case about 75% of the theoretical thrust is being carried by the arch rib proper while 25% is being carried by the superstructure. At the skewbacks the conditions are quite different, the percentage of theoretical thrust carried by the arch rib proper varying from 48 to 81. The variation in skewback thrusts points strongly to the fact that the sliding joints over the pier columns did not

[5] "Application of Freyssinet Method of Concrete Arch Construction," by A. L. Gemeny and C. B. McCullough, M. Am. Soc. C. E., *Technical Bulletin No. 2*, Oregon State Highway Comm., 1933.

[6] *Civil Engineering*, February, 1932, p. 91.

[7] *Proceedings*, Am. Concrete Inst., October, 1932, p. 57.

[8] "A New Electric Telemeter," by Burton McCollom and O. S. Peters, *Technologic Paper No. 247*, National Bureau of Standards, U. S. Dept. of Commerce.

TABLE 5.—PERCENTAGE OF THEORETICAL THRUST OBSERVED FROM TELEMETERS (HORIZONTAL JACKING FORCE, 1 000 000 POUNDS).

Span No.	Percentage at quarter-points	Percentage at skewbacks	Span No.	Percentage at quarter-points	Percentage at skewbacks
1.........	68	67	4.......	72	76
2.........	71	48	6.......	74	72
3.........	78	81	7.......	74	52

function as complete articulations and that the degree of restraint in each case was quite variable.

Therefore, it would appear that, notwithstanding the care exercised to free the arch rib of spandrel restraint to the maximum possible extent, this restraint was great enough to reduce the horizontal crown strains to a value less than one-third the corresponding theoretical values for an unrestrained or free rib, and that, furthermore, the spandrel structure absorbed about 25%

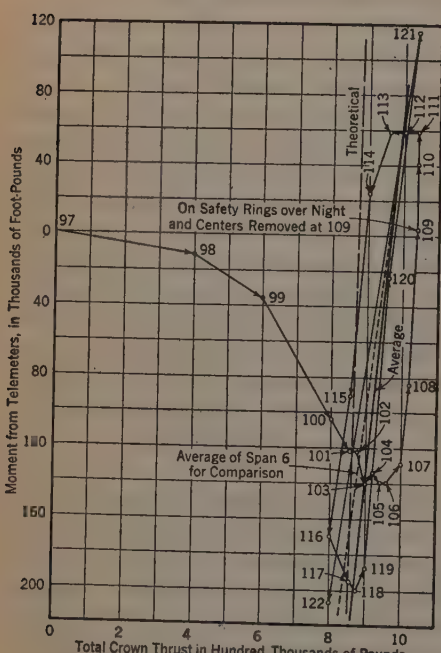

FIG. 21.—TOTAL CROWN THRUST, SPAN NO. 7.

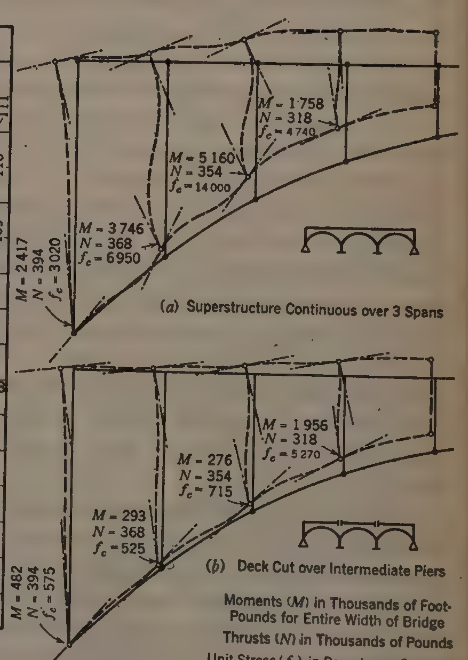

FIG. 22.—MECHANICAL ANALYSIS FOR TEMPERATURE STRESSES IN END SPAN.

of the thrust at the quarter-points and an amount at the skewback which was highly variable because of the varying degree of restraint exercised by the sliding expansion joints over the pier columns.

Another interesting fact in reference to the behavior of the expansion joints and spandrel structure articulations is indicated in Fig. 21 wherein are

plotted the quarter-point moments as determined from telemeter readings against applied crown thrust. Between Readings 107 and 111 it will be noted that the positive moments gradually increase as crown thrusts are increased. Between Readings 111 and 113, however, a decrease of more than 100 000 lb in applied crown thrust produces no change in moment. Below this point, however, as crown pressures are slacked off, quarter-point moments decrease in accordance with theory. The same characteristic lag is indicated at the bottom of the graph between Readings 101 and 107 and also between Readings 116 and 119. Reading 97 (not shown) is at zero moment and thrust.

The same characteristic at the top and bottom of all graphs indicating the relationship between applied crown thrust and arch rib forces and strains at various points persisted throughout the entire test. At first, it was thought that this phenomenon might be explained on the theory of friction in the jacks, but careful calibration and recalibration through a series of test cycles failed to disclose any appreciable jack friction. The only other hypothesis upon which this phenomenon can be explained is that of a frictional resistance to a reversal of motion in the articulations and expansion joints. As long as movement persisted in one direction, superstructure restraint appeared to be more or less uniform and fixed. Whenever motion was reversed, however, there appeared to be sufficient static friction in the articulations to retain the arch in a fixed position, and a considerable increase or decrease in crown thrusts appeared necessary in order to break loose the articulations and permit movement in the reverse direction.

All these results point inevitably to the conclusion that spandrel structure restraint is present to a certain extent even in the most carefully articulated designs; and to the further fact that sliding joints are likely to introduce strain conditions of an exceedingly variant character.

The Rogue River tests were conducted under construction conditions, and certain sources of error were inevitably introduced; however, the persistence, throughout the entire tests, of the results noted herein leaves little room for doubt as to the general tendencies exhibited by the spandrel structure in question. As a result of these researches, it may be logically concluded that the only methods by which the stress distribution in an arch carrying a supported spandrel structure and deck can be determined and controlled accurately, are: (1) By means of a system of articulations much more complete and positive than those ordinarily used heretofore; (2) by means of a system of stress and strain control in the field such as was adopted at the Rogue River Bridge; and (3) by the adoption of a design without expansion joints and a method of calculation that takes into consideration the complete effect of the spandrel structure. In view of the foregoing conclusions, the researches reported by Professor Wilson become exceedingly important.

In connection with the utilization of a spandrel structure without intermediate expansion joints, a word of warning should be sounded against a procedure which carries this idea too far. Fig. 22 is the result of a temperature stress analysis made in the writer's office in 1932. Fig. 22 (*a*) indicates the moments, thrusts, and fiber stresses at the base of spandrel columns supported

by a series of three arch spans, no expansion joints being placed throughout any part of the spandrel structure from end to end. Fig. 22 (*b*) indicates the marked reduction in stress at corresponding points when the deck is severed over the intermediate piers. Even in this latter case the stress at the base of the short column exceeds the ultimate strength of the material, indicating the need for a re-design of the spandrel columns or the provision for further articulation.

All in all, the problem presented is rather complex, and the need for the utmost refinement in analyses of large and important structures becomes increasingly apparent as the problem is further investigated. Structural designers have been altogether too prone to neglect spandrel structure effects and to assume a freedom of arch movement under varying stress conditions which was never realized in actual service. The result in many instances is quite apparent in the formation of cracks at column bases, the opening of construction joints, spalling, and disintegration around expansion joints, and other like conditions. Because of the relief afforded by plastic flow and a redistribution of stress, such conditions may not be sufficient to cause structural failure but, nevertheless, they militate against service life and increase maintenance expense.

In conclusion, the writer wishes again to express his appreciation of the thoroughness and timeliness of Professor Wilson's paper, and the hope that further researches along this line will be stimulated thereby.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

EXPERIMENTS WITH CONCRETE IN TORSION

Discussion

By E. Mirabelli, Assoc. M. Am. Soc. C. E.

E. Mirabelli,[5] Assoc. M. Am. Soc. C. E. (by letter)[5a].—An examination of Fig. 6 reveals that, up to the loading at which the plain concrete specimen fails, the action of a spirally reinforced square specimen is not much different from that of a cylindrical specimen made of plain concrete of the same strength. It may be inferred that reinforcement, when used in moderate quantity, does not take a prominent part in resisting torsion until the tensile strength of the concrete has been overcome. This conclusion is confirmed by test results discussed by Dr. E. Mörsch[8] which show that with spiral reinforcement initial cracks appear on a cylindrical specimen at a load that is only 15% greater than that which produces initial cracking (and simultaneous failure) on a plain concrete cylindrical specimen.

That spiral steel is not fully effective before the concrete yields, may be shown by the following analysis in which concrete is treated as an ideal material, no account is taken of the fact that its modulus of elasticity in tension differs from that in compression, and that stress is not exactly proportional to strain. The notation is the same as that used by the author.

In the spirally reinforced cylindrical member shown in Fig. 10, the steel stress due to torsion is uniform throughout and causes an elongation of the steel which, per unit length of cylinder, is,

$$\Delta_s = \frac{t_s}{E_s} \sqrt{2} \qquad (18)$$

The steel deformation is due to twisting of the cylinder and the point, B, will move to B', making,

$$\overline{BB'} = 2 \frac{t_s}{E_s} \qquad (19)$$

Note.—The paper by Paul Anderson, Assoc. M. Am. Soc. C. E., was published in May, 1934, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[5] Asst. Prof. of Structural Eng., Mass. Inst. Tech., Cambridge, Mass.

[5a] Received by the Secretary May 31, 1934.

[8] "Der Eisenbetonbau," von E. Mörsch, Sixth Edition, Vol. I, Pt. II, Stuttgart, 1929, pp. 303–335.

There will result an angular deformation which, per unit length of cylinder, will be,

$$\psi = \frac{2\, t_s}{E_s\, \rho_s} \quad \text{.........................} (20)$$

Using Equation (1) and substituting $v_e = v_s \dfrac{\rho_e}{\rho_s}$, it is found that,

$$\psi = \frac{v_s}{E'_c\, \rho_s} \quad \text{.........................} (21)$$

From Equations (20) and (21) it is found that,

$$v_s = 2\, \frac{E'_c}{E_s}\, t_s \quad \text{.........................} (22)$$

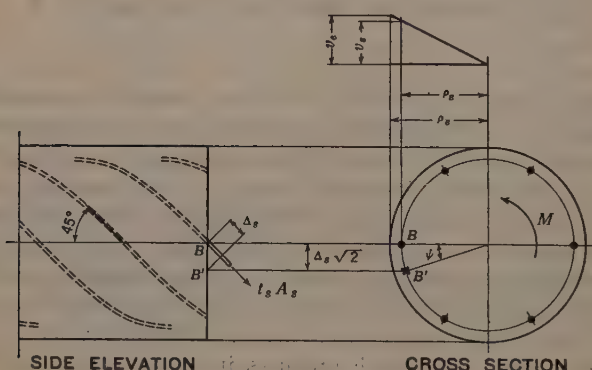

FIG. 10.

Equation (22) expresses the relation between the tension stress, t_s, in the spiral steel, and the torsional shearing stress, v_s, in the concrete adjacent to the steel, in terms of the modulus of elasticity, E_s, for steel and the modulus of rigidity, E'_c, for concrete. Assuming values of $E'_c = 1\,500\,000$ lb per sq in., and $E_s = 30\,000\,000$ lb per sq in., the relation becomes,

$$t_s = 10\, v_s \text{.........................} (23)$$

Neglecting the small reduction in concrete area due to the presence of steel, the part of the torsional resistance provided by the concrete is,

$$M_c = \frac{\pi}{2}\, \rho^3_e\, v_e \quad \text{.........................} (24)$$

and the contribution of the spiral steel is,

$$M_s = N A_s\, t_s\, \frac{1}{\sqrt{2}}\, \rho_s \quad \text{.....................} (25)$$

Assuming 1% of spiral steel and assuming $\rho_s = \dfrac{9}{10}\, \rho_e$, it follows that,

$$M_s = \frac{9}{1\,000\,\sqrt{2}}\, \rho^3_e\, t_s \quad \text{.....................} (26)$$

and from Equations (23), (24), and (26), it is found that,

$$\frac{M_s}{M_c} = 0.1145 \qquad (27)$$

Thus, it is seen that the spiral steel provides about 10% of the total torsional resistance. The percentage resistance will increase in direct proportion to the percentage of steel supplied.

By a similar procedure it may be shown that, for a square member with 45° spiral reinforcement, before the concrete yields and re-distribution of stresses occurs, the relation between tension stress in the steel and shearing stress in the adjacent concrete is approximately,

$$v_s = 2.92 \frac{E'_c}{E_s} t_s \qquad (28)$$

Assuming the same values for E'_c and E_s as for the circular section,

$$t_s = 6.85\ v_s \qquad (29)$$

and for 1% spiral steel,

$$\frac{M_s}{M_c} = 0.0628 \qquad (30)$$

In this case the spiral steel provides about 6% of the total torsional resistance. Here, again, the percentage resistance will increase with the percentage of steel supplied.

The preceding anaylsis leads to a consideration of the deduction that (see "Conclusions"), "* * * the spiral reinforcement can be assumed to take practically all tensile stresses in excess of the ultimate tensile strength of the unreinforced concrete." This statement may be interpreted to mean that after the working stress in torsional shear for plain concrete has been exhausted, the remainder of the torsional moment, regardless of its magnitude, may be assigned to the spiral steel at a safe working stress. If the concrete working stress in torsional shear is taken as 150 lb per sq in., it follows from Equation (23) that, for a circular section, the corresponding steel stress is only 1 500 lb per sq in. Such a unit stress will result in a wasteful use of steel. If the steel is stressed to 18 000 lb per sq in., the adjacent concrete will tend to be stressed to about 1 800 lb per sq in., but will crack before this stress is reached, and only a small core of material will still be effective in diagonal tension.

It is reasonable to assume that the factor of safety to be used in design should be based on the loading that causes cracking rather than on that which produces ultimate failure even if, because of the presence of steel, the member shows considrable toughness and strength after cracking begins. Cracks are objectionable and, as indicated by Fig. 6, when the concrete begins to yield (at about one-half the ultimate strength of the member) there is a rapid increase in the rate of twist. From Table 2, it appears that a working stress of 150 lb per sq in. in torsion provides a factor of safety of about two for plain concrete, with an ultimate compressive strength of 2 250 lb. per sq in.

A safe, economical, and consistent method would be to design the member as if it were of plain concrete, using a limiting working stress of about 150

lb per sq in. (for 2 250-lb. concrete). For the purpose of increasing the factor of safety, enough spiral steel area may be provided (at 15 000 lb per sq in.) to resist the total design moment, assuming the concrete to have cracked and the steel to be acting alone. For a circular section, using $v_e = 150$ lb per sq in., $t_s = 15\,000$ lb per sq in., and $\rho_s = \frac{9}{10}\,\rho_e$, it is found that a total steel area of 0.79% of the total cross-section area is sufficient for this purpose. For a square section the corresponding figure is 0.66 per cent. If the torsional shear is 50 lb per sq in., or less, there will be a sufficient margin of safety without the use of reinforcement.

It may be noted that in a cylindrical member the maximum stress intensity occurs at the surface and is uniform throughout, whereas in a member of rectangular section the maximum stress occurs on a line along the middle of the long side. There is opportunity for considerable re-distribution of stress in a member of rectangular section before cracking occurs. For this reason a somewhat higher working stress intensity (about 200 lb per sq in. for 2 250-lb. concrete) might be used for a rectangular section than for a circular section.

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DISCUSSIONS

FLEXIBLE "FIRST-STORY" CONSTRUCTION FOR EARTHQUAKE RESISTANCE

Discussion

BY HOWARD G. SMITS, ESQ.

HOWARD G. SMITS,[11] ESQ. (by letter)[11a].—The analysis developed by Mr. Green is interesting; it is quite a new thought as applied to the problem of duplicating, mathematically, such complex vibrations, as earthquake oscillations. Undoubtedly, complex earth motions can be approximated by this method of description more closely than by the usual periodic sine curve or cosine curve expressions.

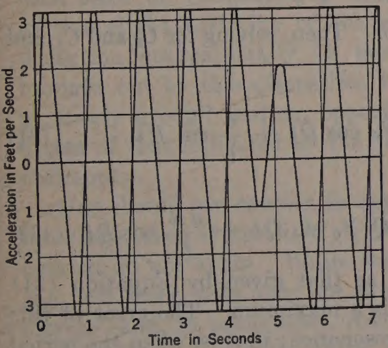

FIG. 11.—ACCELERATION DIAGRAM.

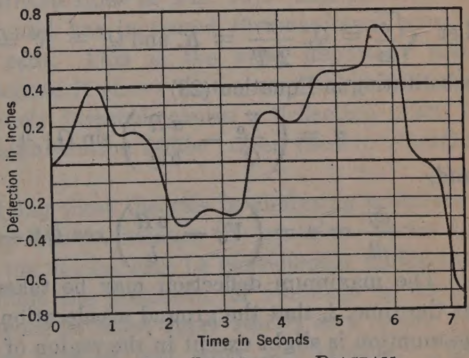

FIG. 12.—DEFLECTION DIAGRAM.

It is to be noted, however, that the same general method of utilizing unequal maximum accelerations may be used with the customary harmonic theory. Substituting Fig. 11 for Mr. Green's acceleration diagram (Fig. 3), it will be seen that, in general, the periods and accelerations are the same in the two cases with the exception that with the curved diagram the acceleration is zero when $t = 0$. At any given time the ground is moving with simple harmonic motion, such that the acceleration of the ground is $a \sin \frac{\pi}{2T} t$.

NOTE.—The paper by Norman B. Green, Esq., was published in February, 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: In May, 1934, by Messrs. Lee H. Johnson, Edward J. Bednarski, and Merit P. White and Paul L. Kartzke.

[11] Chf. Designing Engr. for Oliver G. Bowen, Los Angeles, Calif.

[11a] Received by the Secretary April 11, 1934.

Using Mr. Green's nomenclature the differential equation for the horizontal deflection of the tops of the columns from their bases is:

$$m \frac{d^2 y}{dt^2} = m\,a \sin \frac{\pi}{2\,T}\,t - e\,y \quad \text{.................} (27)$$

and the general solution of this equation is:

$$y = C_3 \sin \sqrt{\frac{e}{m}}\,t + C_4 \cos \sqrt{\frac{e}{m}}\,t + \frac{a}{\frac{e}{m} - \frac{\pi_2}{4\,T^2}} \sin \frac{\pi}{2\,T}\,t \quad \text{....} (28)$$

in which, a = maximum acceleration, and T = time for acceleration to decrease from the maximum to zero (one-fourth the full period of oscillation).

Two constants of integration, C_3 and C_4, are involved which are similar to C_1 and C_2 in Mr. Green's analysis. These constants are evaluated from the initial conditions, namely, when $t = 0, \frac{dy}{dt} = V_0$; $y = Y_0$; and,

$$Y_0 = C_4 \quad \text{.............................} (29)$$

and,

$$V_0 = C_3 \sqrt{\frac{e}{m}} + \frac{\frac{a\,\pi}{2\,T}}{\frac{e}{m} - \frac{\pi^2}{4\,T^2}} \quad \text{.....................} (30)$$

Let $\sqrt{\frac{e}{m}} = Q$, $\frac{\pi}{2\,T} = R$, and $Q^2 - R^2 = b$. Then, solving for C_3 and C_4 and substituting in Equation (28):

$$y = \left(\frac{V_0}{Q} - \frac{a\,R}{b\,Q}\right) \sin Q\,t + Y_0 \cos Q\,t + \frac{a}{b} \sin R\,t \quad \text{......} (31)$$

and,

$$\frac{dy}{dt} = v = \left(V_0 - \frac{a\,R}{b}\right) \cos Q\,t - Q\;Y_0 \sin Q\,t + \frac{a\,R}{b} \cos R\,t \quad \text{..} (32)$$

The maximum deflection may be taken as that given by Equation (31) at the time, t, that the ground acceleration is a maximum. The error of this assumption is slight except in the region of resonance; that is, when the period of the ground approaches the free period of the columns.

Mr. Green might have included damping in his discussion. Damping, which is caused by the absorption of energy by the building, may be taken as proportional to the first power of the velocity. If p is taken as the proportionality constant, Equation (1) becomes:

$$m \frac{d^2 y}{dt^2} + p \frac{dy}{dt} = m\,a\left(1 - \frac{t}{T}\right) - e\,y \quad \text{............} (33)$$

and the general solution of this linear equation is:

$$y = e^{-j} \left[C_5 \sin \sqrt{\frac{e}{m} - \left(\frac{p}{2\,m}\right)^2} + C_6 \cos \sqrt{\frac{e}{m} - \left(\frac{p}{2\,m}\right)^2} \right] + \frac{m\,a}{e}\left[1 + \frac{p}{e\,T} - \frac{t}{T}\right] \quad \text{.......................} (34)$$

in which $j = \frac{p}{2\,m}\,t$; and C_s and C_e are the constants of integration and are determined by the initial conditions. Actually, damping is an important consideration; but it is not pertinent to the conclusions of this discussion.

Equations (31) and (32) may be applied in much the same manner as Equations (3), (4) and (5). Table 2 shows the computations for this set of conditions. The horizontal deflection of the tops of the columns from their bases is shown as it varies with time in Fig. 12.

TABLE 2.—COMPUTATIONS

Duration from beginning of motion, t_a	Duration of uniform periodic motion, t_p	T	a	INITIAL CONDITIONS		R	b	FINAL CONDITIONS	
				Y_0	V_0			Y_n	V_n
4.50	4.50	0.250	3.2	0	0	6.29	—36.4	+0.312	+0.525
4.83	0.33	0.165	1.0	+0.312	+0.525	10.40	—105.0	+0.453	+0.121
5.50	0.67	0.335	2.0	+0.453	+0.121	4.70	—19.0	+0.502	—0.392
7.25	1.75	0.250	3.2	+0.502	—0.392	6.29	—36.4	—0.718	—0.157

Conclusions.—If the ground had continued oscillating with the same motion with which it began, the maximum possible deflection would have been 0.402 in. (indicated by the broken lines in Fig. 12). The introduction of this slight variation of the ground has increased the maximum possible deflection approximately 79 per cent. This is the most important point brought out by this discussion, because it shows that a very slight variation of steady ground motion, opportunely injected, causes a deflection much in excess of that indicated by the "magnification factor" often used in vibration philosophy.

One should use caution in applying these theories in analyzing multi-story buildings, even if the first story is flexible, because the equations under discussion do not apply. While the top stories may be considerably stiffer than the first story, this does not preclude the possibility of vibration in the top stories—and, in some cases, resonance in these upper stories at a higher frequency than that with which the base or first story is vibrating. These vibrations of the upper stories materially affect the motions of the first story. The conclusions given are applicable only to a one-story bent and not to a multi-story building.